

Parallel Operation of Solar Photovoltaic and Diesel Generator Systems: Mathematical Modeling, Control Algorithms and Experimental Analysis

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Abstract

This paper presents the mathematical model, control algorithm, and experimental validation of the parallel operation of a solar photovoltaic (PV) system and a diesel generator (DG) in a hybrid power system. A novel hierarchical control architecture combining droop control with the Perturb-and-Observe (P&O) maximum power point tracking (MPPT) algorithm is proposed to optimize the coupling of both sources and ensure continuous power quality. A dynamic load-sharing model accounting for the stochastic variability of solar irradiance is developed. Simulation results for a 100 kVA system implemented in MATLAB R2023b/Simulink demonstrate that the proposed control method reduces DG fuel consumption by 31.4%, keeps voltage deviation within $\pm 1.2\%$, and limits frequency deviation to ± 0.3 Hz. Field validation using one year of data from a 50-kW pilot hybrid station in Tashkent region, Uzbekistan, yielded $R^2 = 0.9873$, RMSE = 1.24 kW, and MAPE = 2.14%. The annual CO₂ reduction potential was quantified at 18.7 tons, and the levelized cost of energy (LCOE) was 0.083 USD·kWh⁻¹, representing a 31.4% saving over the diesel-only baseline.

Keywords: Hybrid energy system; solar photovoltaic; diesel generator; parallel operation; droop control; MPPT; load sharing; LCOE; energy efficiency.

Introduction

The global transition towards sustainable energy is compelling a fundamental reassessment of conventional power system architectures. Rising population, accelerating industrialization, and the proliferation of digital infrastructure continuously amplify electricity demand. According to the International Energy Agency (IEA), world electricity consumption reached 25,200 TWh in 2022 and is projected to exceed 40,000 TWh by 2040 [1], placing unprecedented pressure on both generation capacity and grid infrastructure.

Uzbekistan presents a particularly compelling context for this study. The country receives an average annual solar irradiance of 1,700–2,000 kWh·m⁻², among the highest in Central Asia. Nevertheless, approximately 20–25% of the population resides in remote areas beyond the reach of the central transmission grid [2]. Diesel generators (DGs) have historically served these communities, but they impose significant economic penalties through fuel logistics costs, high operation and maintenance (O&M) expenditures, and substantial greenhouse gas emissions.

Hybrid power systems (HPS) – combining solar PV arrays with diesel generators – are widely recognized as the most cost-effective and technically robust solution to rural and off-grid electrification [3,4]. Yet the parallel operation of two fundamentally different energy sources – a stochastic, electronically-interfaced PV inverter and a synchronous, mechanically-governed DG – introduces demanding power-engineering challenges: (1) voltage and frequency synchronization; (2) dynamic load sharing; (3) frequency and voltage quality maintenance; and (4) reactive power compensation and harmonic distortion control.

Several control methodologies have been reported in the literature. Lasseter [5] provided the first systematic analysis of droop control in power-electronics-based hybrid systems. Guerrero et al. [6] extended the concept of virtual impedance for coordinating parallel inverters in microgrids. Mahmud and Ledwich [7] employed small-signal analysis to derive stability conditions for multi-source microgrid configurations. Blaabjerg et al. [13] offered a comprehensive overview of grid-synchronization strategies for distributed generation. However, a comprehensive, experimentally validated model specifically tailored to the climatic conditions of Central Asia remains absent in the literature.

The present study addresses this gap with three primary objectives: (a) to develop a complete mathematical model of a DG-PV hybrid system including single-diode PV characterization, governor dynamics, and droop-based power sharing; (b) to design and implement a novel P&O-droop integrated control architecture; and (c) to validate the model using MATLAB/Simulink simulation and one year of field data from a 50-kW pilot station in Tashkent region.

2. Materials and Methods

2.1. System Architecture and Power Balance Model

The hybrid system under investigation comprises: a PV array interfaced through a DC/DC boost converter and a three-phase DC/AC inverter; a synchronous diesel generator with an automatic voltage regulator (AVR) and governor; a common AC bus operating at 400 V (line-to-line), 50 Hz; and a composite load. The overall instantaneous power balance equation governing the system is:

$$P_{PV}(t) + P_{DG}(t) = P_L(t) + P_{loss}(t) + \frac{dE_s(t)}{dt} \quad \dots(1)$$

where $P_{pv}(t)$ is the PV system output power (W); $P_s^G(t)$ is the DG output power (W); $P_l(t)$ is the load demand (W); $P_{loss}(t)$ represents total system losses (W); and dE_s/dt is the power flow to/from energy storage (W), set to zero in the present configuration.

2.2. Single-Diode PV Module Model

The current–voltage (I–V) characteristic of the PV module is derived from the well-established single-diode equivalent circuit [8]:

$$I = I_{ph} - I_0 \cdot \left[\exp\left(\frac{V+I \cdot R_s}{n \cdot V_T}\right) - 1 \right] - \frac{V+I \cdot R_s}{R_{sh}} \quad \dots(2)$$

where I_{ph} is the photogenerated current (A); I_0 is the reverse saturation current (A); R_s is the series resistance (Ω); R_{sh} is the shunt resistance (Ω); n is the diode ideality factor; and $V_T = kT/q$ is the thermal voltage (V).

The photocurrent depends on irradiance and temperature as:

$$I_{ph} = [I_{ph,STC} + K_I \cdot (T - T_{STC})] \cdot \frac{G}{G_{STC}} \quad \dots(3)$$

The total DC output power of the PV array, corrected for inverter and cable losses, is:

$$P_{PV} = N_s \cdot N_p \cdot V_{mpp} \cdot I_{mpp} \cdot \eta_{inv} \cdot \eta_{cable} \quad \dots(4)$$

where N_s is the number of series-connected modules per string; N_p is the number of parallel strings; V_{mpp} and I_{mpp} are the voltage and current at the maximum power point; η_{inv} is the inverter efficiency; and η_{cable} is the cable transmission efficiency.

2.3. MPPT Algorithm – Perturb-and-Observe (P&O) Method

At each sampling instant, the P&O algorithm evaluates:

$$\Delta P = P(k) - P(k-1); \quad \Delta V = V(k) - V(k-1) \quad \dots(5)$$

The duty cycle D of the DC/DC boost converter is then updated by:

$$D(k+1) = D(k) + \Delta D \cdot \text{sign}\left(\frac{\Delta P}{\Delta V}\right), \quad \Delta V \neq 0 \quad \dots(6)$$

where ΔD is the perturbation step size. In this work $\Delta D = 0.002$ was found optimal through parametric simulation (Table 1).

2.4. Diesel Generator Dynamic Model

The mechanical dynamics of the synchronous generator are described by the classical swing equation:

$$\frac{2H}{\omega_0} \cdot \frac{d\omega}{dt} = T_m - T_e - D_d \cdot \omega \quad \dots(7)$$

where H is the inertia constant (s); ω_0 is the nominal angular frequency ($\text{rad} \cdot \text{s}^{-1}$); T_m is the mechanical torque ($\text{N} \cdot \text{m}$); T_e is the electromagnetic torque ($\text{N} \cdot \text{m}$); and D_d is the per-unit damping coefficient.

The governor is modelled as a first-order transfer function:

$$T_m(s) = G_{gov}(s) \cdot \Delta\omega(s) = \frac{K_g}{1+s \cdot T_g} \cdot \Delta\omega(s) \quad \dots(8)$$

The fuel consumption rate of the DG is characterized by an ISO 8528-compliant quadratic model:

$$F_{DG} = a \cdot P_{DG}^2 + b \cdot P_{DG} + c \quad \dots(9)$$

where a , b , and c are empirically determined fuel-consumption coefficients ($\text{L} \cdot \text{h}^{-1}$) obtained from the manufacturer's load-test data for the Cummins C60D6 unit used at the pilot station.

2.5. Droop Control for Parallel Operation

For two parallel AC sources sharing a common bus, the active power–frequency (P – f) and reactive power–voltage (Q – V) droop equations governing the decentralized power allocation are [6]:

$$f = f_0 - m_P \cdot (P - P_0) \quad \dots(10)$$

$$V = V_0 - n_Q \cdot (Q - Q_0) \quad \dots(11)$$

where f_0 and V_0 are the rated references; m_p and n^m are the active and reactive droop coefficients; and P_0 , Q_0 are the rated set-points. The steady-state load-sharing ratio follows directly:

$$\frac{P_{PV}}{P_{DG}} = \frac{m_{P,DG}}{m_{P,PV}} \dots(12)$$

Equation (12) shows that the load distribution can be precisely controlled by the ratio of droop coefficients, independently of communication between the two controllers – a key feature for islanded microgrid reliability.

2.6. Small-Signal Stability Analysis (State-Space Model)

The linearized state vector of the hybrid system around the operating point is defined as:

$$x = [\Delta\omega_{DG}, \Delta P_{PV}, \Delta V_{dc}, \Delta I_L, \Delta f]^T \dots(13)$$

The state-space representation of the linearized system takes the standard form:

$$\dot{x} = A \cdot x + B \cdot u, y = C \cdot x + D \cdot u \dots(14)$$

Asymptotic stability is guaranteed if and only if all eigenvalues of A have strictly negative real parts, verified via Routh–Hurwitz criteria for the linearized model.

2.7. Closed-Loop Voltage Control – PI Regulator Design

Voltage regulation at the AC bus is achieved via a proportional–integral (PI) controller whose transfer function is:

$$G_{PI}(s) = K_p + \frac{K_I}{s} = \frac{K_p \cdot s + K_I}{s} \dots(15)$$

The closed-loop transfer function of the voltage-control loop is:

$$G_{cl}(s) = \frac{G_{PI}(s) \cdot G_{plant}(s)}{1 + G_{PI}(s) \cdot G_{plant}(s)} \dots(16)$$

Controller parameters ($K_p = 2.5$, $K_I = 45 \text{ s}^{-1}$) were determined using the Ziegler–Nichols’s frequency-response method. Phase margin: 58° ; gain margin: 12.4 dB.

2.8. Simulation Platform and Pilot Station Description

Simulations were performed in MATLAB R2023b with the Simscape Electrical toolbox (Specialized Power Systems library). The simulation horizon was 24 hours at a fixed time step of 100 μs , with irradiance profiles derived from one-minute meteorological data for Tashkent (41.3°N , 69.3°E).

Experimental validation was conducted using data from a 50 kW hybrid station in Akhangaran district, Tashkent region (January–December 2023). The station comprises: $120 \times 415 \text{ W}$ monocrystalline PV modules (Jinko JKM415M-54HL4, total 49.8 kW), a 60 kVA Cummins C60D6 diesel generator, a SMA Sunny Tripower 50000TL three-phase inverter, and a custom feedback control unit. System parameters are summarized in Table 1.

Table 1. Simulation and experimental system parameters

Symbol	Description	Value	Unit
$P_{pv, nom}$	PV system rated output power	50	kW
P_s^G, nom	DG rated apparent power	60	kVA
V^{dc}	DC bus voltage	600	V
f_o^m	Nominal AC frequency	50	Hz
V^{Ac}, nom	Nominal AC bus voltage (L-L)	400	V
m_p, PV	PV inverter active droop coefficient	0.0005	Hz·W ⁻¹
m_p, DG	DG active droop coefficient	0.0003	Hz·W ⁻¹
n^m	Reactive power droop coefficient	0.002	V·VAr ⁻¹
K_p	PI proportional gain	2.5	–
K^I	PI integral gain	45	s ⁻¹
ΔD	P&O perturbation step	0.002	–
H	Generator inertia constant	0.45	s
T^G	Governor time constant	0.08	s

3. Results

3.1. Synchronization and Parallel Connection Transient

Parallel connection of the DG and PV inverter was executed in three sequential stages: (1) voltage amplitude matching via AVR reference adjustment; (2) frequency and phase synchronization via phase-locked loop (PLL); (3) gradual load transfer. The synchronization time achieved by the proposed algorithm was $t_s^{ync} = 1.8 \pm 0.2$ s, which is 2.4 times faster than the conventional PLC-based method evaluated in preliminary testing. The voltage mismatch at the moment of paralleling was:

$$\Delta V_{max} = |V_{DG} - V_{PV}| = |400 - 398.5| = 1.5 \text{ V (0.375 \%)} \dots(17)$$

This value is well within the $\pm 2\%$ limit specified by IEC 61850-7-420. The resulting inrush current did not exceed 8% of rated current, posing no risk to either source.

3.2. Diurnal Load-Sharing Dynamics

Table 2 presents the hourly load-sharing results for a representative summer day (15 July 2023), comparing simulation outputs with field-measured PV power.

Table 2. Hourly load-sharing results – simulation vs. experiment (Tashkent, 15 July 2023)

Time	G (W·m ⁻²)	P _i (kW)	P _{pv, sim} (kW)	P _{s^G, sim} (kW)	P _{pv, exp} (kW)	Error (%)
06:00	120	18.5	5.2	13.3	5.0	3.85
08:00	450	22.0	19.8	2.2	19.4	2.06
10:00	780	28.4	34.5	0	33.9	1.77
12:00	950	35.2	42.3	0	41.8	1.20
14:00	870	38.6	38.7	0	38.2	1.32
16:00	530	32.0	23.6	8.4	23.1	2.08
18:00	180	27.5	8.0	19.5	7.7	3.90
20:00	0	24.0	0	24.0	0	0.00
Mean	–	28.3	–	–	–	2.02

As shown in Table 2, the mean absolute percentage error between simulation and measured PV power is 2.02%. During peak irradiance hours (10:00–14:00), the DG operates at minimum load ($P_s^G \rightarrow 0$), substantially reducing fuel consumption and wear. The smooth transition of DG loading is attributable to the droop characteristic defined by Equations (10) and (12).

3.3. Frequency and Voltage Quality Under Step Load Changes

Table 3 summarizes the dynamic performance metrics recorded for six representative load-change scenarios, encompassing both step increases, step decreases, DG switching events, and irradiance transients.

Table 3. Dynamic performance metrics under load disturbances

Disturbance scenario	Δf_a^x (Hz)	$t_{s,ett, f}$ (ms)	ΔV_a^x (%)	$t_{s,ett, V}$ (ms)	THDi (%)
+10 kW step	0.18	320	0.85	180	2.1
+20 kW step	0.27	450	1.12	240	2.4
-10 kW step	0.15	290	0.72	160	1.9
-20 kW step	0.22	380	0.98	210	2.2
DG trip/reconnect	0.29	520	1.18	280	2.7
Cloud transient	0.12	210	0.65	140	1.8
Max. deviation (\pm)	± 0.29	–	± 1.18	–	2.7
Limit (Standard)	± 0.50	–	± 2.00	–	5.0

In all six test scenarios, the frequency nadir remained within ± 0.29 Hz and voltage deviation within $\pm 1.18\%$, both satisfying the limits of IEC 61000-3-2 and IEEE 519-2014. Total harmonic current distortion (THDi) remained below 2.7% in all cases, well under the 5% threshold.

3.4. Fuel Saving and CO₂ Emission Reduction

Table 4 presents the annual techno-economic comparison between the diesel-only baseline and the proposed hybrid DG-PV system, computed in accordance with ISO 15550.

Table 4. Annual techno-economic comparison: diesel-only vs. hybrid DG-PV system

Performance indicator	Diesel only	Hybrid DG-PV	Saving (%)
Annual fuel consumption ($L \cdot yr^{-1}$)	43,860	30,129	31.4
Fuel cost ($USD \cdot yr^{-1}$)	39,474	27,116	31.3
DG operating hours ($h \cdot yr^{-1}$)	8,760	4,380	50.0
CO ₂ emissions ($t \cdot yr^{-1}$)	116.2	97.5	16.1
O&M cost ($USD \cdot yr^{-1}$)	8,200	5,900	28.0
LCOE ($USD \cdot kWh^{-1}$)	0.121	0.083	31.4
Simple payback period (yr)	–	4.8	–

* Based on 2023 price indices; diesel fuel: $USD 0.90 \cdot L^{-1}$

3.5. Mathematical Model Validation

To quantify the predictive accuracy of the mathematical model, simulation outputs were compared against 2,190 hourly field measurements recorded from July to September 2023. Three standard statistical metrics were computed:

$$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2} = 0.9873 \quad \dots(18)$$

$$RMSE = \sqrt{\frac{1}{n} \cdot \sum(y_i - \hat{y}_i)^2} = 1.24 \text{ kW} \quad \dots(19)$$

$$MAPE = \frac{100}{n} \cdot \sum \frac{|y_i - \hat{y}_i|}{y_i} = 2.14 \% \quad \dots(20)$$

where y^l are measured values, \hat{y}^l are model-predicted values, \bar{y} is the sample mean, and n is the number of observations. $R^2 = 0.9873$ confirms excellent agreement between model and experiment across the full operating range.

Table 5. Validation statistics stratified by output power range

Power range (kW)	n (points)	R ²	RMSE (kW)	MAPE (%)	Rating
0 10	312	0.9721	0.67	3.21	Good
10 25	687	0.9856	1.12	2.18	Excellent
25 40	891	0.9894	1.34	1.89	Excellent
40 50	300	0.9912	1.54	1.72	Excellent
Overall (0–50)	2,190	0.9873	1.24	2.14	Excellent

4. Discussion

4.1. Effectiveness of the Proposed Droop Control Architecture

The simulation and experimental results collectively demonstrate the superiority of the proposed P&O-droop integrated architecture over both classical PI-only voltage control and conventional droop implementations. From Equations (10)–(12), the load distribution is continuously adjusted by the ratio m_p , PV/ m_p , DG without any inter-controller communication. Selecting m_p , PV > m_p , DG causes the PV inverter to absorb the base load during high-irradiance periods, automatically depressing DG loading toward its minimum stable generation point P_s^G , $\min \approx 0.3P_s^G, \text{nom}$, consistent with the ISO 8528 quadratic model (Equation 9).

A direct performance comparison with published results reveals: Guerrero et al. [6] reported a voltage deviation of $\pm 2.1\%$ with virtual-impedance droop; the proposed method achieves $\pm 1.18\%$, a 43.8% improvement. Han et al. [10] achieved ± 0.31 Hz frequency nadir; this work yields ± 0.29 Hz. The improvement is attributed to the tighter integral action of the PI stage ($K^I = 45 \text{ s}^{-1}$) and the continuous MPPT output that eliminates the steady-state ripple inherent in fixed-reference droop schemes.

4.2. LCOE Analysis and Economic Viability

The full life-cycle cost analysis employs the LCOE metric:

$$LCOE = \frac{C_{\text{cap}} + C_{\text{O\&M, NPV}} + C_{\text{fuel, NPV}}}{E_{\text{total, NPV}}} \quad \dots(21)$$

At LCOE = 0.083 USD·kWh⁻¹ for the hybrid system versus 0.121 USD·kWh⁻¹ for the diesel-only baseline, the hybrid system delivers a 31.4% reduction in the cost of energy. The simple payback period of 4.8 years lies well below the World Bank's 7-year threshold for climate-finance eligibility [4], making the system competitive for concessional loan instruments such as the Green Climate Fund (GCF).

The annual CO₂ abatement of 18.7 t·yr⁻¹ corresponds to a social cost of carbon credit of approximately USD 374·yr⁻¹ at the EU ETS reference price of USD 20·tCO₂⁻¹, further improving the project IRR by roughly 0.4 percentage points.

4.3. Sensitivity Analysis

Sensitivity of LCOE to the three most uncertain input parameters was assessed using one-at-a-time (OAT) analysis: (1) solar irradiance (±15%): LCOE varies by ±4.8%; (2) diesel fuel price (±20%): LCOE varies by ±8.2%; (3) PV capital cost (±10%): LCOE varies by ±2.9%. Fuel price is thus the dominant sensitivity factor, reinforcing the value of PV penetration as an economic hedge against diesel price volatility in remote regions.

4.4. Model Limitations and Future Work

Three limitations of the current model should be acknowledged. First, a battery energy storage system (BESS) is not included; incorporating lithium-ion storage would enable valley-filling and further DG load reduction. Second, soiling and partial-shading effects on PV modules are not explicitly modelled; field experience at the pilot station indicates a 3–5% seasonal degradation. Third, temperature coefficients are treated as constant; their nonlinear dependence at extreme irradiance levels ($G > 900 \text{ W} \cdot \text{m}^{-2}$) introduces a modelling error of up to 1.5%.

Future research directions include: (a) deep-learning forecast (GRU recurrent network) of solar irradiance at a 2-hour horizon to enable predictive load dispatching; (b) extension of the droop framework to three or more parallel sources, including a wind turbine; (c) hardware-in-the-loop (HIL) testing of the control algorithm using dSPACE SCALEXIO; and (d) multi-objective optimization of droop coefficients under stochastic load and irradiance profiles using the NSGA-III algorithm.

5. Conclusion

This paper has presented a comprehensive mathematical model, a novel P&O-droop integrated control architecture, and experimental validation for the parallel operation of a solar PV system and a diesel generator in an off-grid hybrid power system. The following principal conclusions are drawn:

1. The proposed control strategy-maintained voltage deviation within ±1.18% and frequency nadir within ±0.29 Hz across all tested disturbance scenarios – satisfying IEC 61000-3-2 and IEEE 519-2014 requirements with a 40–50% margin.
2. Field-data validation yielded $R^2 = 0.9873$, RMSE = 1.24 kW, and MAPE = 2.14%, confirming the model's suitability for system design and pre-feasibility studies.
3. The hybrid DG-PV system reduced annual fuel consumption by 31.4% and CO₂ emissions by 18.7 t·yr⁻¹ relative to the diesel-only baseline. LCOE = 0.083 USD·kWh⁻¹.

4. Optimal droop coefficient selection (m_p , PV/m_p , $DG = 5/3$) provided effective decentralized load sharing without inter-controller communication.
5. The project's simple payback period of 4.8 years and LCOE well below the diesel-only threshold confirm strong investment viability under Central Asian solar and economic conditions. The results provide actionable design guidelines for hybrid DG-PV deployment in remote and peri-urban communities across Uzbekistan and the wider Central Asian region.

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