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Soft $\mathfrak{M}$ -Separation Axioms in Dual Soft Topological Space

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Abstract
Here in this work we will introduce a new type of sparation axioms using "soft dual local function" where obtained . This type of space will be studied in terms of its properties and its results will be proven and their interrelationship with other theories.

Keywords: second type of soft local function, $\mathfrak{M}$ ^(*∘)-space, $\mathfrak{M}$ ^(∘)-space, $\mathfrak{M}$ ^(***∘)-space ans $\mathfrak{M}$ ^(****∘)-space.**

1. Introduction

Shannen introduced a new genre called R0 topological spaces [1] in 1943. And then Davis [2] found and developed the weak separation axioms. sundry topologists (e.g. [3], [4], [5]) Dontchv and Puartes 2008 [6] consolidated several weak separation particular of topological ideal. Noiri and Kaskin 2011 [7] They developed ideal topological spaces in finding Λ -sets and relied on them to construct new types of weak separation axioms. Al-Swide and AL-Nefae, 2013 [8] worked on separation axiom over "Gem Set ". Hussain and Ahmad [9] take the soft points to build soft separation axioms in soft topological spaces and Thus, they obtained a set of results. Kandil et al. [10] respectively, and they investigated Soft semi separation axioms and soft functions and their distinctive [10] studied Soft semi separation axioms and some types of soft functions and their characteristics . In this work , we introduce new definition M-separation Axioms via benefit of second type of soft local function and study relationship between them.

2. Preliminaries

Definition 2.1[11]: The sub collection $II_{(U_1,U_2)}$ of $DS_{(U_1,U_2)}$is called the dual soft ideal if satisfy the following.

1. If $(A, F_1, G_1) \in II_{(U_1,U_2)_E}$ and $(B, F_2, G_2)_E \subseteq (A, F_1, G_1)$ then$(B, F_2, G_2)_E \in II_{(U_1,U_2)_E}$.

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2. If (A, F_1, G_1). (B, F_2, G_2) $\in II_{(U_1,U_2)_E}$ then (A, F_1, G_1) $\cup$ (B, F_2, G_2) $\in II_{(U_1,U_2)_E}$.

Definition 2.2[11]: The sub collection StT_E of $DS_{(U_1,U_2)_E}$ is called dual soft topology StT_Eon X_{Ds} if satisfy.

1. $\emptyset_{Ds}$, $X_{Ds} \in StT_E$.
2. If FG_A,$f_1G_{1_B} \in StT_E$ then $FG_A \cap f_1G_{1_B} \in StT_E$.
3. For any index Λ if $f_iG_{i_{A_I}} \in StT_E$. Then $\cup_{i\in\Lambda} f_iG_{i_{A_I}} \in StT_E$.

So StT_E= $\{\emptyset_{Ds}$, $X_{Ds}, FG_A\}$ is dual soft topology and (X_{Ds}, StT_E) is dual soft topological space

Definition 2.3 [12]: Let ($X_{Ds}, StT_E, II_{(U_1,U_2)}$) be dual soft ideal topological space and FG_Abe a subset of X_{Ds}. Then the forma.

$FG_A{}^*$= $\{e_{xy} \in X_{Ds}$ for each $HK_A \in StT_E(e_{xy}) : HK_A \cap FG_A \notin II_{(U_1,U_2)}\}$ is called second type of soft local function of FG_A, when HK_A is StT_E-open and $e_{xy} \in StT_E$.

Definition 2.4:- Let (X_{Ds}, StT_E) be a dual soft ideal topological space , $FG_A \subseteq X_{Ds}$ we define $pr^{*e_{xy}}(FG_A)$= $FG_A{}^* \cup FG_A$ for each $e_{xy} \in X_{Ds}$.

Proposition2.5 :-Let FG_A be such sets of (X_{Ds}, StT_E), $e_{xy} \in X_{Ds}$. Then $pr^{*e_{xy}}(FG_A) \subseteq$cl($FG_A$)

Definition 2.6:- A sub set FG_A of a dual soft ideal topological space (X_{Ds}, StT_E) is called dual dual perfected set if $FG_A{}^* \subseteq FG_A$.

Definition2.7 :-A dual soft ideal topological space (X_{Ds}, StT_E) is called dual outright space if ∀ subset of X_{Ds} is dual outrighted set .

Definition2.8 :-Let **(**X_{Ds}, StT_E)be a dual soft ideal topological space.For each $e_{xy} \in X_{Ds}$, a nonempty subset FG_A of X_{Ds}, is called a dual Strong set if and if $FG_A{}^*$ is open and $e_{xy} \in FG_A$.

Definition2.9 :- A dual soft ideal topological space (X_{Ds}, StT_E) is said $k_\circ$ -space if and only if for each open set FG_A and $e_{xy} \in FG_A$ implies cl{ $cl\{(e_{xy}, \emptyset, \emptyset)\} \subseteq FG_A$.

Definition2.10 :-A dual soft ideal topological space**(**(X_{Ds}, StT_E) is said dual dual Strong space if every sub set of X_{Ds} is a dual Strong set.

Proposition2.11 :- Let (X_{Ds}, StT_E) be a dual soft ideal topological space a dual outright subset FG_A of X_{Ds} is open ,If X_{Ds} is a dual Strong space .

3. Some Soft $\mathfrak{M}$ -Separation Axioms in dual soft topological space

Definition3.1: A dual soft ideal topological space (X_{Ds}, StT_E) is said to be

1. $\mathfrak{M}^{*\circ}$-space iff ∀ dual strong setFG_A and $e_{xy} \in FG_A$ implies $pr^{*e_{xy}}\{(e_{xy}, \emptyset, \emptyset)\} \subseteq FG_A{}^*$
2. $\mathfrak{M}^{**\circ}$-space iff ∀ open set FG_A and $e_{xy} \in FG_A$ implies $pr^{*e_{xy}}\{(e_{xy}, \emptyset, \emptyset)\} \subseteq FG_A{}^*$
3. $\mathfrak{M}^{***\circ}$-space iff ∀ a sub set FG_A of X_{Ds}, and $e_{xy} \in FG_A$ implies $cl\{(e_{xy}, \emptyset, \emptyset)\} \subseteq FG_A{}^*$

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4. $\mathfrak{M}^{***\circ}$-space iff $\forall$ a dual outright set FG_A , and $e_{xy} \in FG_A$ implies $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl(FG_A)$

Theorem3.2 : Every $\mathfrak{M}^{***\circ}$-space is $\mathfrak{M}^{*\circ}$-space .

Proof: Let $e_{xy} \in X_{Ds}$,and FG_A be dual strong set of X_{Ds}be an $\mathfrak{M}^{***\circ}$-space $cl\{(e_{xy},\emptyset,\emptyset)\} \subseteq {FG_A}^*$ by proposition (2.5) $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl\{(e_{xy},\emptyset,\emptyset)\} \subseteq {FG_A}^*$, then $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq {FG_A}^*.It$ follows that X_{Ds} is $\mathfrak{M}^{*\circ}$-space.

Theorem 3.3:Every $\mathfrak{M}^{***\circ}$-Space is $\mathfrak{M}^{****\circ}$-Space.

Proof:Let $e_{xy} \in X_{Ds}$ and FG_A be dual iff $\forall$ se tof $\mathfrak{M}^{***\circ}$-space X_{Ds} $\ni$ Let $e_{xy} \in FG_A$ then $cl\{(e_{xy},\emptyset,\emptyset)\} \subseteq {FG_A}^*$ so by proposition (2.5) we have that $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl\{(e_{xy},\emptyset,\emptyset)\} \subseteq {FG_A}^* \subseteq cl\{FG_A\}$, then $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl\{FG_A\}$, , it follows that X is **$\mathfrak{M}^{****\circ}$-space.**

Proposition3.4: If $(X_{Ds}, StT_E)$is outright space and dual Strong space . Then every $m^{***\circ}$-space is $m^{**\circ}$-space .

Proof :

Let $e_{xy} \in X_{Ds}$, FG_A be an open subset of X_{Ds} such that $e_{xy} \in FG_A$ .But X_{Ds} is then FG_A *is* dual outrighted set implies that ${FG_A}^* \subseteq FG_A$.Since X_{Ds} is **$\mathfrak{M}^{***\circ}$-space then** $cl\{(e_{xy},\emptyset,\emptyset)\} \subseteq {FG_A}^*$ by proposition (2.5) then $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl\{(e_{xy},\emptyset,\emptyset)\}$, it follows that $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq FG_A$ .Hence X_{Ds} is a $\mathfrak{M}^{**\circ}$-space.

Proposition3.5: If (X_{Ds}, StT_E) is a dual Strong space . Then every $\mathfrak{M}^{***\circ}$-space is $\mathfrak{M}^{****\circ}$-space.

Proof :-Let $e_{xy} \in X_{Ds}$, FG_A be dual outright set of X_{Ds} suchthat $e_{xy} \in FG_A$ .Since X_{Ds} is dual strong space by proposition (2.11) then FG_A is open .But X_{Ds} is $\mathfrak{M}^{**\circ}$-space we have that $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq {FG_A}^* \subseteq cl(FG_A)$ then $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl(FG_A)$ it follows that X_{Ds} is $\mathfrak{M}^{****\circ}$-space**.**

Proposition3.6: Every $k_\circ$ -space is $\mathfrak{M}^{**\circ}$-space**.**

Proof:

Let $e_{xy} \in X_{Ds}$ and FG_A be an open such that $e_{xy} \in FG_A$ .Since X_{Ds} is $k_\circ$ -space then $e_{xy} \in FG_A$ and $cl\{(e_{xy},\emptyset,\emptyset)\} \subseteq FG_A$,by proposition(1.9) so we have that $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl\{(e_{xy},\emptyset,\emptyset)\} \subseteq FG_A$ then $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq FG_A$ .Hence X is **$\mathfrak{M}^{**\circ}$-space.**

Proposition3.7: if (X_{Ds}, StT_E) is dual Strong space . Then every $k_\circ$ -space is $\mathfrak{M}^{****\circ}$-space.

Proof :-Let $e_{xy} \in X_{Ds}$ and FG_A be dual outrighted set of X_{Ds} such that $e_{xy} \in FG_A$,since X_{Ds} is dual Strong space by proposition (2.11) then FG_A is open set .But X_{Ds} is $k_\circ$ **-space, then** $cl\{(e_{xy},\emptyset,\emptyset)\} \subseteq FG_A$ so by proposition(2.5) so we have that $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl(FG_A)$, then $pr^{*e_{xy}}\{(e_{xy},\emptyset,\emptyset)\} \subseteq cl(FG_A)$. It follows that X_{Ds} is **$\mathfrak{M}^{****\circ}$-space.**

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Proposition 3.8: If $((X_{Ds}, StT_E)$ is outrighted space and dual strong space , then every $\mathfrak{M}^{***\circ}$-space is $k_\circ$ -space.

Proof :

Let FG_A open set and $e_{xy} \in X_{Ds}$ such that $e_{xy} \in FG_A$,since X is $\mathfrak{M}^{***\circ}$ -Space then $cl\{(e_{xy}, \emptyset, \emptyset)\} \subseteq FG_A{}^*$. But X_{Ds} is dual outrighted space then FG_A is is dual outrighted set implies that $FG_A{}^* \subseteq FG_A$, ot follows that $cl\{(e_{xy}, \emptyset, \emptyset)\} \subseteq FG_A$.Then X_{Ds} is a $k_\circ$ -space**.**

Theorem 3.9: A dual soft ideal topological space (X_{Ds}, StT_E) is is $\mathfrak{M}^{***\circ}$-space if and if for each $e_{xy} \in X_{Ds}$ and FG_A sub set of X_{Ds} with $cl\{(e_{xy}, \emptyset, \emptyset)\}^* \subseteq FG_A{}^*$.,where $e_{xy} \in FG_A$.

Proof :

Let $e_{xy}. \in X_{Ds}$ containing e_{xy} Since X_{Ds} is $\mathfrak{M}^{***\circ}$ -space then $cl\{(e_{xy}, \emptyset, \emptyset)\} \subseteq FG_A{}^*$ .But $(e_{xy}, \emptyset, \emptyset)^* \subseteq cl\{(e_{xy}, \emptyset, \emptyset)\} \subseteq cl(\, cl\{(e_{xy}, \emptyset, \emptyset)\})$ it follows $cl\{(e_{xy}, \emptyset, \emptyset)\}^* \subseteq cl\{(e_{xy}, \emptyset, \emptyset)\}) \subseteq FG_A{}^*$.

Conversely Let $e_{xy} \in FG_A$, but $\{(e_{xy}, \emptyset, \emptyset)\} \subseteq cl\{(e_{xy}, \emptyset, \emptyset)\}^* \subseteq FG_A{}^*$. Therefor (X_{Ds}, StT_E) is $\mathfrak{M}^{**\circ}$-space

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