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Graph Functions in Terms of "Gem-Sets"

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Abstract. The primary purpose of paper is to define and study a novel type of graph function under the idea of " Gem-Set" named β^*-closed graph , ω^*-closed graph and 〖sem〗^*-closed graph , then we define K^*-map , 〖KO〗^*-map , T^*-map and T^(**)-map , then we are study the properties of it's and the relationships with our function .

Keywords: closed graph functions, K^*-map , 〖KO〗^*-map , J^*-map and J^(**)-map.

1. Introduction

In the product space, each continuous function from a topological space to a T_2-space has closed graph. This symmetry of a continuous function with its graph allows one to view it as an element from the space "of closed subsets of the product space and" consequently to award the space of "continuous functions C(X,Y) with the relative topology of set- convergence of Choquet [2] and" Kuratowski [4].In 2020 Rana , Reem & Hairan[8] introduced the $\boldsymbol{\theta}$-Continuity in Micro Topological Spaces**.** It is known that, for a function f : R → R with a closed graph, a sufficient and necessary condition for being continuous is the connectedness of the graph Burgess [1]. They said to g is closed graph if graph of g is the set {(z, g(z)) : z ∈ Z} is a closed subset of the product space $Z \times Y$. g is strongly closed graph if $z \in Z, y \in Y$, $y=g(z)$ infers there exist two open set R and H contained z and y such that $g(R) \cap cl\, H = \emptyset$ L.L. Herrington [5]. We use the idea of "Gem Set" to define ore closed graph function. That is, put (Z, τ) is a topological space , $A \subseteq Z, z \in Z$, then A^{*z} with regard to (Z, τ) as follows : $A^{*z} = \{y \in Z : G \cap A \notin I_z, for\ every\ G \in T(y)\}\ where\ T(y) = \{G \in T : y \in G\}$, A set A^{*z} is called "Gem-Set" AL-Swidi & AL-Nafee [6]. An ideal for a one point and denoted by I_x [5] , the ideal on a topological space (Z, τ) at point z is well-defined by $I_z = \{V \subseteq Z : z \in V^c\}$, where V is a non-empty subset of Z .

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2. Preliminaries

Recall that the graph function : Let $g:(Z,\tau)\longrightarrow(Y,\sigma)$. Graph function of g is *the subset* of $Z\times Y$ defined by $G(g)=\{(z,g(z));z\in Z\}$. The ideal to z denoted by I_z [4] , An ideal I_z on a topological space (Z,τ) at point z is defined by $I_z=\{V\subseteq Z : z\in V^c\}$, where V is a non-empty subset of Z .

Definition 2.1 [6]: The mapping h : (Z , τ) $\longrightarrow$ (R , ρ) is named:

- $I^*-map\ \ iff\ \forall\ K\subseteq Z, z\in Z\ , h(K^{*z})=(h(K))^{*h(z)}$.
- $I^{**}-map\ \ iff\ \forall\ K\subseteq R\ , r\in R\ , h^{-1}(K^{*r})=(h^{-1}(K))^{*h^{-1}(r)}$.

Definition 2.2 [7] : The mapping $h\ :\ (Z,\tau)\ \longrightarrow\ (R,\rho)$ is named:

- h is $A-map$ at $z\in Z\ \ iff\ \ \forall\ L\subseteq R\ ,\exists\ K\subseteq Z\ \ s.t\ h(K^{*z})\subseteq L^{*h(z)}$.
- h is $AO-map\ \ iff\ \forall\ K\subseteq Z\ ,\exists\ L\subseteq R\ \ s.t\ \ L^{*r}\subseteq h[K^{*h^{-1}(r)}]$.

Definition 2.3 [6]: (Z ,τ) is a topological space , K ⊆Z , K is called prefected set if $K^{*z}\subseteq K$, for each $z\in Z$.

Definition 2.4 [3]: The function $h\ :\ Z\ \rightarrow R$ is said to have a strongly closed graph *iff* for each $z\in Z$ and each $r\in R$ such that $r\neq h(z)$, there exist an open subset U containing z in Z and open subset V containing r in $R\ s.t\ (U\times\bar{V})\cap G(h)=\emptyset$.

Definition 2.5 [3]: The function $h\ :\ Z\ \rightarrow R$ is said to have a θ-strongly closed graph *iff* for each $z\in Z$ and each $r\in R$ such that $r\neq h(z)$, ∃ open subset U containing $z\ in\ Z$ and open subset V containing $r\ in\ R\ \ s.t\ \ (\bar{U}\times V)\cap G(h)=\emptyset$.

Definition 2.6 [6]: (Z ,τ) is a topological space called $I^*-T_2\ space\ iff\ \ \forall\ z\neq r$ of Z, $\exists\ A\ ,B\ \neq\emptyset$ of Z $s.t\ \ \ A^{*z}\cap B^{*r}=\emptyset, with\ r\notin A^{*z}\ and\ z\notin B^{*r}$.

Definition 2.7 [6] : (Z ,τ) is a topological space called a strongly -space (briefly s-space) *iff* $\forall\ A\neq\emptyset\ ,\ \ A\subseteq X$ is *strongly* subset

Proposition 2.8 [6]: Let (Z ,τ) be a topological space , and let A and B be a subsets of Z and $z\in Z$. Then $A^{*z}\subset\bar{A}$.

Proposition 2.9 [6]: Let (Z ,τ) be a topological space , and let A and B be a subsets of Z and $z\in Z$. Then If $z\in Z$. Then $z\in A$ if and only if $z\in A^{*z}$.

3. The novel graph function and properties of it

We present in this section , a novel graph function with the “Gem-Set” , that is to say , β^*-closed graph , β^{**}-closed graph , ω^*-closed , s^*-closed graph , s^{**}-closed graph and A^*, AO^*, T^*, T^{**} - maps , In addition to studying its properties and its relationship with the compacted space .

Definition3.1 : *Let h : (Z ,τ)* $\longrightarrow$ *(R ,ρ)* **,** We tell the function that it is:

1. β^*- closed graph $iff\ \forall z\in Z\ ,r\in R\ ,r\neq h(r)$ there exists a subset K in Z containing z and subset L in R containing r , $s.t\ (K\times{}^{*r}pr(L))\cap G(h)=\emptyset$.
2. $\beta^{**}-$ closed graph $iff\ \forall z\in Z\ ,r\in R\ ,r\neq h(r)$ there exists an open subset K in Z containing z and open subset L in R containing r , s.t $(K\times{}^{*r}pr(L))\cap G(h)=\emptyset$.

Proposition 3.2 : The function h from Z topological space *into* R prefected topological space, h has β^{**}-closed graph function iff for each $z\in Z$ and each $r\in R\ s.t\ \ r\neq h(z)$ there exists open subset K containing z in Z and open subset L containing r in R , s.t $h(K)\cap L=\emptyset$.

Proof : Let $z\in Z$ and each $r\in R$ such that $r\neq h(z)$. Since $G(h)$ is β^{**}-closed so there exist open $K\subseteq Z$, $z\in K$ and open $L\subseteq R$, $r\in L$, $h(K)\cap{}^{*r}pr(L)=\emptyset$, by hypothesis R is prefected space so $L={}^{*r}pr(L)$. So we get that $h(K)\cap L=\emptyset$.

Conversely : Let $z \in Z$ and each $r \in R$ such that $r \neq h(z)$, $\exists$ $open\ K \subseteq Z$, $z \in K$ and open $L \subseteq R$, $r \in L$. By hypothesis $h(K) \cap L = \emptyset$, and R is prefected space so $L = {}^{*r}pr(L)$. So we get that $h(K) \cap {}^{*r}pr(L) = \emptyset$. Thus h has β^{**}-closed graph function .

Proposition 3.3 : The function h from Z topological space *into* R prefected topological space , h has β^*-closed graph function iff for each $z \in Z$ and each $r \in R$ $s.t$ $r \neq h(z)$, $\exists$ a subset K in Z , $z \in K$ and subset L in R , $r \in L$, $s.t$ $h(K) \cap L = \emptyset$.

Proof : The proof by the same way of above proposition .

Definition 3.4 : Let $h : (Z, \tau) \longrightarrow (R, \rho)$, then h has $\omega^* - closed\ graph\ iff$ $\forall z \in Z$, $r \in R$, $r \neq h(z)$ there exists subset $K \subseteq Z$ containing z and $\exists$ open $L \subseteq R$ containing r , $s.t$ $(K^{*z} \times L) \cap G(h) = \emptyset$.

Proposition 3.5 : The function h from Z topological space into R topological space , h has $\omega^* - closed\ graph$ if $\forall\ z \in Z$, $r \in R$ $S.t.$ $r \neq h(z)$, $\exists$ $K \subseteq Z\ containing\ z\ and\ \exists\ open\ L \subseteq R\ containing\ r$, $s.t.$ $h(\bar{K}) \cap L = \emptyset$.

Proof : Let $z \in Z$ and each $r \in R$ $S.t. r \neq h(z)$, $\exists$ $K \subseteq Z\ containing\ z\ and\ \exists\ open\ L \subseteq R\ containing\ r$. By hypothesis $h(\bar{K}) \cap L = \emptyset$ and by proposition (2.8) so $K^{*z} \subseteq \bar{K}$. So we get that $h(K^{*z}) \subseteq h(\bar{K})$. Therefore $h(K^{*z}) \cap L = \emptyset$. Thus h has $\omega^* - closed\ graph$ function .

Definition 3.6 : Let $h : (Z, \tau) \longrightarrow (R, \rho)$ **,** We tell the function that it is:

1. $sem^* - closed\ graph$ iff $\forall z \in Z$, $r \in R$, $r \neq h(z)$, $\exists$ $K \subseteq Z\ containing\ z\ and\ \exists$ $L \subseteq R$ containing r , S.t. $(K^{*z} \times L^{*r}) \cap G(h) = \emptyset$.
2. sem^{**} – closed graph iff $\forall z \in Z$, $r \in R$, $r \neq h(z)$, $\exists$ open $K \subseteq Z\ containing\ z\ and\ \exists\ open\ L \subseteq R$ containing r , S.t. $(K^{*z} \times L^{*r}) \cap G(h) = \emptyset$.

Proposition 3.7 : The function h from Z topological space into R topological space , h has sem^* – closed if $\forall\ z \in Z$, $r \in R$ $S.t.$ $r \neq h(z)$, $\exists$ $K \subseteq Z\ containing\ z\ and\ \exists\ L \subseteq R\ containing\ r$, $s.t.$ $h(\bar{K}) \cap \bar{L} = \emptyset$.

Proof : Let $z \in Z$ and each $r \in R$ $S.t. r \neq h(z)$, $\exists$ $K \subseteq Z\ containing\ z\ and\ \exists\ L \subseteq R\ containing\ r$. By hypothesis . $h(\bar{K}) \cap \bar{L} = \emptyset$ and by proposition (2.8) so $K^{*z} \subseteq \bar{K}$ and $L^{*r} \subseteq \bar{L}$. So we get that $h(K^{*z}) \subseteq h(\bar{K})$. Therefore $h(L) \cap L^{*r} = \emptyset$. Thus h has sem^* – closed graph function .

Proposition 3.8 : The function h from Z topological space into R topological space , h has sem^{**} – closed if $\forall\ z \in Z$, $r \in R$ $S.t.$ $r \neq h(z)$, $\exists$ $open\ K \subseteq Z\ containing\ z\ and\ \exists\ open\ L \subseteq R\ containing\ r$, $s.t.$ $h(\bar{K}) \cap \bar{L} = \emptyset$

Proof : The proof by the same way of above proposition .

Note 3.9 : Let $h : (Z, \tau) \longrightarrow (R, \rho)$, for any subset V of Z we define $q(V)$ as $q(V) = V \times h(V)$, where q is a function from (Z, τ) to $(Z \times R, \tau \times \sigma)$.

Definition 3.10 : Let $h : (Z, \tau) \longrightarrow (R, \rho)$ and $q: (Z, \tau) \longrightarrow (Z \times R, \tau \times \sigma)$:

1. we say that q is K^*-map if $q(V^{*z}) \subseteq (V \times h(V))^{*(z,h(z))}$, for each $V \subseteq Z$ and $z \in Z$.
2. we say that q is KO^* -map if $(V \times h(V))^{*(z,h(z))} \subseteq G(V^{*z})$, for each $V \subseteq Z\ and\ z \in Z$.
3. we say that q is J^*-map if $q(V^{*Z}) = V^{*z} \times [h(V)]^{*h(z)}$, for each $V \subseteq Z\ and\ z \in Z$.

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4. we say that q is J^{**}-map if$q\left(B^{*y} \times h^{-1}(B^{*r})\right) = (h^{-1}(B))^{*h^{-1}(r)}$, for each $B \subseteq R$ and $r \in R$.

Note 3.11 : We let $q:(Z,\tau) \longrightarrow (Z \times R, \tau \times \sigma)$ then we get that : Always $V^{*z} \times B^{*r} \subseteq (V \times B)^{*(z,r)}$.

Proof : Let $(c,d) \in (V^{*z} \times B^{*r})$ such that $c \in V^{*z}$ and $d \in B^{*r}$. Then $K_c \cap V \notin I_z$,$\forall K_c \in \tau$ and $L_d \cap B \notin I_r$,$\forall L_d \in \sigma$. So $K_c \times L_d \in \tau \times \sigma$ .Hence $(c,d) \in K_c \times L_d \in \tau \times \sigma$. So we get that $K_c \times L_d \cap V \times B \notin I_{(z,h(z))}$. Therefore $(c,d) \in (V \times B)^{*(z,r)}$. Thus $V^{*z} \times B^{*r} \subseteq (V \times B)^{*(z,r)}$.

Theorem 3.12 : Let $h : (Z,\tau) \longrightarrow (R,\rho)$, $q:(Z,\tau) \longrightarrow (Z \times R, \tau \times \sigma)$ is the graph function of h then : If h is A-map then q is K^*-map .

Proof : Let h is A-map , to prove that q is K^*-map . Let $V \times h(V) \subseteq Z \times R$ with $V \subseteq Z$, $h(V) \subseteq R$. Since h is A-map so $h(V^{*z}) \subseteq h(V)^{*h(z)}$. Then $G(V^{*z}) = V^{*z} \times h(V^{*z}) \subseteq V^{*z} \times h(V)^{*h(z)}$ $\subseteq (V \times h(V))^{*(z,h(z))}$, by note (3.11) . Therefore $q(V^{*z}) \subseteq (q(V))^{*(z,h(z))}$. Thus q is K^*-map .

Theorem 3.13 : Let $h : (Z,\tau) \longrightarrow (R,\rho)$, $q:(Z,\tau) \longrightarrow (Z \times R, \tau \times \sigma)$. If q is KO^*-map then h is AO-map.

Proof : Let q is KO^*-map, to prove that h is AO-map . Let $V \subseteq Z$, $z \in Z$, since q is AO^*-map , we have $(q(V))^{*(z,h(z))} \subseteq q(V^{*z})$ iff $(V \times h(V))^{*(z,h(z))} \subseteq q(V^{*z} \times h(V^{*z}))$, now by use note (3.11) we get that $V^{*z} \times h(V)^{*h(z)} \subseteq (V \times h(V))^{*(z,h(z))} \subseteq V^{*z} \times h(V^{*z})$.

Therefore $h(V)^{*h(z)} \subseteq h(V^{*z})$. Thus h is AO-map .

Theorem 3.14 : Let $h : (Z,\tau) \longrightarrow (R,\rho)$, $q:(Z,\tau) \longrightarrow (Z \times R, \tau \times \sigma)$, then : h is I^*-map iff q is J^*-map.

Proof : Let h is I^*-map to prove that q is J^*-map . Let $V \subseteq Z$, it follows that $q(V^{*z}) = V^{*z} \times h(V^{*z})$, Since h is I^*-map , so $h(V^{*z}) = [h(v)]^{*h(z)}$, which implies that $q(V^{*z}) = V^{*z} \times [h(v)]^{*h(z)}$. Thus q is J^*-map .

Conversely : Let $V \subseteq Z$, since q is J^*-map , which is implies that $q(V^{*z}) = V^{*z} \times [h(v)]^{*h(z)}$, then , we have $V^{*z} \times h(V^{*z}) = V^{*z} \times [h(v)]^{*h(z)}$. Therefore $h(V^{*z}) = [h(v)]^{*h(z)}$. Thus$h$ is I^*-map .

Theorem 3.15 : Let $h : (Z,\tau) \longrightarrow (R,\rho)$, $q:(Z,\tau) \longrightarrow (Z \times R, \tau \times \sigma)$, then : h is I^{**}-map iff q is J^{**}-map.

Proof : Let $B \subseteq R$ and $r \in R$, so we have that $q(h^{-1}(B^{*r}) \times B^{*r}) = h^{-1}(B^{*r})$, since h is I^{**}-map , it follows that $h^{-1}(B^{*r}) = (h^{-1}(B))^{*h^{-1}(r)}$ .Therefore $q(h^{-1}(B^{*r}) \times B^{*r}) = (h^{-1}(B))^{*h^{-1}(r)}$. Thus q is J^{**}-map .

Conversely : To prove that h is I^{**}-map . Let $B \subseteq R$ and $r \in R$ so by assumption we have $q(h^{-1}(B^{*r}) \times B^{*r}) = (h^{-1}(B))^{*h^{-1}(r)}$. But $q(h^{-1}(B^{*r}) \times B^{*r}) = h^{-1}(B^{*r})$, then we get that $h^{-1}(B^{*r}) = (h^{-1}(B))^{*h^{-1}(r)}$ .Thus h is I^{**}-map.

Theorem 3.16 : The function h from (Z,τ) topological space into (R,ρ) topological space and $q:(Z,\tau) \longrightarrow (Z \times R, \tau \times \sigma)$, if q is J^*-map then q is K^*-map .

Proof : Let q is J^*-map , to prove that q is K^*-map . iff $V \subseteq Z$and $z \in Z$, since q is J^*-map , so we get that $q(V^{*z}) = V^{*z} \times [h(v)]^{*h(z)}$.

$\subseteq (V \times h(V))^{*(z,h(z))}$, By note (3.11). Thus q is K^*-map .

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Corollary 3.17 : If h is I^*-map , then q is K^*-map .

Proof : Let h is I^*-map , then by theorem (3.14) we have q is J^*-map , and by above theorem (3.16) implies that q is K^*-map .

Remark 3.18 : the following diagram holds :

$h\ :\ (Z\,,\tau)\ \longrightarrow\ (\ R\,,\rho\ )$ and $G(h)$ is the graph function of h, then :

J	K^*-map graph function

3.1. Diagram : The relationship between J^*-map and K^*-map graph function .

Theorem 3.19 : The function h from $(Z\,,\tau)$ topological space into $(\ R\,,\rho\ )\,prefected$ topological space , h is injective mapping, if h has ω^*-closed graph function then h has sem^{**}-closed graph function .

Proof : Let $z\in Z$ and each $r\in R$ such that $r\neq h(r)$. By hypothesis h has ω^*-closed graph function . So $\exists\ \ K\subseteq Z\ containing\ z\ and\ \exists\ open\ L\subseteq R\ containing\ r$ S.t. $(K^{*z}\times L)\ \cap\ G(h)=\emptyset$. Ever since R is prefected space then $L^{*r}\subseteq L$. So we get $(K^{*z}\times L^{*r})\ \cap\ G(h)=\emptyset$. Thus h has sem^*-closed graph function.

Theorem 3.20 : The function h from $(Z\,,\tau)\,prefected$ topological space into $(\ R\,,\rho\ )$ topological space , h is injective mapping, if h has $\beta^{**}-$closed graph function then :

i. h is sem^*-closed graph function .

ii.h is $sem^{**}-$closed graph function .

iii.h is ω^*-closed graph function .

Proof :

i.Let $z\in Z$ and each $r\in R$ such that $r\neq h(z)$. By hypothesis h has $\beta^{**}-$closed graph function . So $\exists\ open\ \ K\subseteq Z\ containing\ z\ and\ \exists\ open\ L\subseteq R\ containing\ r$ S.t. $(K\times{}^{*r}pr(L))\ \cap\ G(h)=\emptyset$. But Z is prefected space , then $K^{*z}\subseteq K$. So we get that $(K^{*z}\times L^{*r})\ \cap\ G(h)=\emptyset$. Thus h has sem^*-closed graph function .

ii. The proof by the same way of proposition (2.8) .

iii. Let $z\in Z$ and each $r\in R$ such that $r\neq h(z)$. By hypothesis h has $\beta^{**}-$closed graph function . So $\exists\ open\ \ K\subseteq Z\ containing\ z\ and\ \exists\ open\ L\subseteq R\ containing\ r$ S.t. $(K\times{}^{*r}pr(L))\ \cap\ G(h)=\emptyset$. Since Z is prefected space then $K^{*z}\subseteq K$, and we know that $L\subseteq{}^{*r}pr(L)$. Hence we get that $(K^{*z}\times L)\ \cap\ G(h)=\emptyset$. Thus h has ω^*-closed graph function .

Theorem 3.21 : The function h from $(Z\,,\tau)$ topological space into $(\ R\,,\rho\ )$ topological space , h is injective mapping , if h is $\beta^{**}-$closed graph function then h is β^*-closed graph function .

Proof : Let $z\in Z$ and each $r\in R$ such that $r\neq h(z)$. By hypothesis h has $\beta^{**}-$closed graph function . So $\exists\ open\ \ K\subseteq Z\ containing\ z\ and\ \exists\ open\ L\subseteq R\ containing\ r$ S.t. $(K\times{}^{*r}pr(L))\ \cap\ G(h)=\emptyset$. . Thus h has β^*-closed graph function .

Theorem 3.22 : The function h from $(Z\,,\tau)\,prefected$ topological space into $(\ R\,,\rho\ )\,prefected$ topological space , h is injective mapping, if h is β^*-closed graph function then h is sem^*-closed graph function .

Proof : Let $z\in Z$ and each $r\in R$ such that $r\neq h(z)$. By hypothesis h is β^*-closed graph function . So $\exists\ \ K\subseteq Z\ containing\ z\ and\ \exists\ \ L\subseteq R\ containing\ r$ S.t. $(K\times{}^{*r}pr(L))\ \cap$

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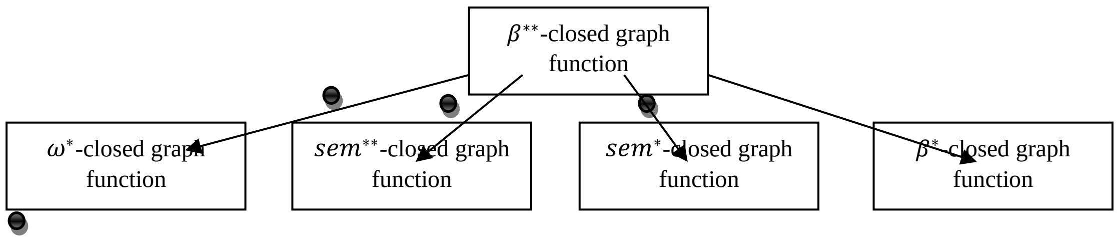
$G(h) = \emptyset$. But Z is prefected so we get that $K^{*z} \subseteq K$. Therefore $(K^{*z} \times L^{*r}) \cap G(h) = \emptyset$. Thus h is $sem^* -$ closed graph function .

Theorem3.23 : The function h from (Z , τ) topological space into (R , ρ) $prefected$ topological space , h is injective mapping, if h is $sem^{**} -$ closed graph function then h is $sem^* -$ closed graph function .

Proof : Let $z \in Z$ and each $r \in R$ such that $r \neq h(z)$. By hypothesis h is $sem^{**} -$ closed graph function . So $\exists$ $open\ K \subseteq Z\ containing\ z\ and\ \exists\ open\ L \subseteq R\ containing\ r$ S.t. $(K^{*z} \times L^{*r}) \cap G(h) = \emptyset$. Thus h is $sem^* -$ closed graph function .

Remark 3.24 : the following diagram holds . $h : (Z , \tau) \longrightarrow (R , \rho)$ then :



Z is prefected space .

R is prefected space .

3.2 Diagram : The relationship between $\beta^* -$ closed , $\beta^{**} -$ closed , $\omega^* -$ closed , $sem^* -$ closed and $sem^{**} -$ closed graph function.

Theorem 3.25 : The function h from (Z , τ) topological space into (R , ρ) topological space , h is injective mapping, if h is a *strongly closed graph* function then f is $\beta^{**} -$ closed [resp. $\beta^* -$ closed] graph function .

Proof : Let $z \in Z$ and each $r \in R$ such that $r \neq h(z)$. By hypothesis h is *strongly closed graph* function . So $\exists$ $open\ K \subseteq Z\ containing\ z\ and\ \exists\ open\ L \subseteq R\ containing\ r$ S.t. $(K \times \bar{L}) \cap G(h) = \emptyset$. By proposition (2.9) we have ${}^{*r}pr(L) \subseteq \bar{L}$. So we get that $(K \times {}^{*r}pr(L)) \cap G(h) = \emptyset$. Thus h has $\beta^{**} -$ closed [resp. $\beta^* -$ closed] graph function **.**

Theorem 3.26 : The function h from (Z , τ) topological space into (R , ρ) $perfect$ topological space , h is injective mapping, if h has $\theta -$ strongly closed graph function then h has $\beta^* -$ closed [resp. $\beta^{**} -$ closed] graph function .

Proof : Let $z \in Z$ and each $r \in R$ such that $r \neq h(z)$. By hypothesis f has $\theta -$ strongly closed graph function . So $\exists$ $open\ K \subseteq Z\ containing\ z\ and\ \exists\ open\ L \subseteq R\ containing\ r$ S.t. $(\bar{K} \times L) \cap G(h) = \emptyset$. we know that $K \subseteq \bar{K}$. And by hypothesis R is prefected space so ${}^{*r}pr(L) \subseteq L$. So we get that $(K \times {}^{*r}pr(L)) \cap G(h) = \emptyset$. Thus f has $\beta^* -$ closed [resp. $\beta^{**} -$ closed] graph function .

Theorem 3.27 : The function h from (Z , τ) $perfect$ topological space into (R , ρ) topological space , h is injective mapping, if h is a *strongly closed graph* function then h has $\omega^* -$ closed graph function .

Proof : Let $z \in Z$ and each $r \in R$ such that $r \neq h(z)$. By hypothesis h has *strongly closed graph* function . So $\exists$ $open\ K \subseteq Z\ containing\ z\ and\ \exists\ open\ L \subseteq R\ containing\ r$ S.t. $(K \times \bar{L}) \cap G(h) = \emptyset$. Since Z is prefected space we'll get $K^{*z} \subseteq K$, and we have $L \subseteq \bar{L}$. we'll get $(K^{*z} \times L) \cap G(h) = \emptyset$. Thus h has $\omega^* -$ closed graph function .

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Theorem 3.28 : The function h from (Z,τ) topological space into (R,ρ) topological space , h is injective mapping, if h has $\theta-$strongly closed graph function then h has ω^*-closed graph function .

Proof : Let $z\in Z$ and each $r\in R$ such that $r\neq h(z)$. By hypothesis h has $\theta-$strongly closed graph function . So $\exists\ open\ K\subseteq Z\ containing\ z\ and\ \exists\ open\ L\subseteq R\ containing\ r$ S.t. $(\bar{K}\times L)\cap G(h)=\emptyset$. By proposition (2.8) we've got $K^{*z}\subseteq\bar{K}$. we'll get $(K^{*z}\times L)\cap G(h)=\emptyset$. Thus h has ω^*-closed graph function .

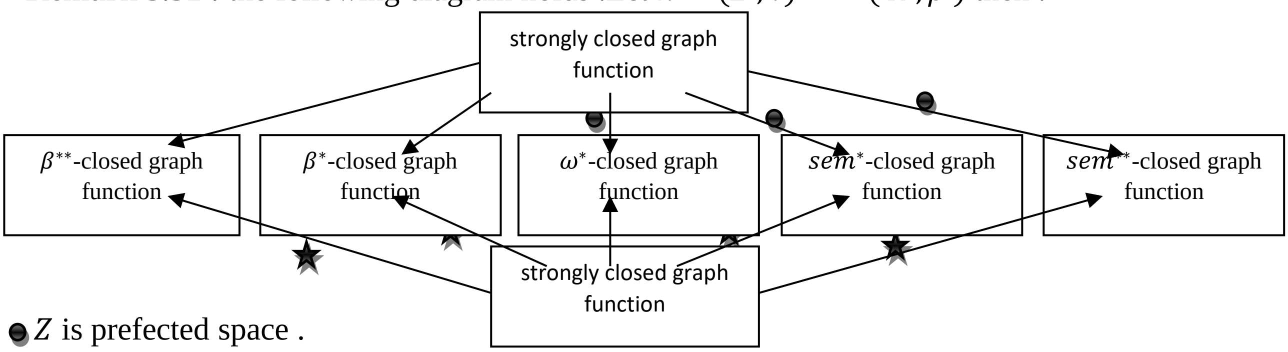
Theorem 3.29 : The function h from (Z,τ) $perfect$ topological space into (R,ρ) topological space , h is injective mapping, if h is *strongly closed graph* function, then h is sem^*-closed [resp. $sem^{**}-$closed] graph function .

Proof : Let $z\in Z$ and each $r\in R$ such that $r\neq h(z)$. By hypothesis h has *strongly closed graph* function . So $\exists\ open\ K\subseteq Z\ containing\ z\ and\ \exists\ open\ L\subseteq R\ containing\ r$ S.t. $(K\times\bar{L})\cap G(h)=\emptyset$. By proposition (2.8) we've got $L^{*r}\subseteq\bar{L}$. And by hypothesis Z is *perfected space* so $K^{*z}\subseteq K$. we'll get $(K^{*z}\times L^{*r})\cap G(h)=\emptyset$. Thus h has sem^*-closed [resp. $sem^{**}-$closed] graph function .

Theorem 3.30 : The function h from (Z,τ) topological space into (R,ρ) $perfect$ topological space , h is injective mapping,, if h has $\theta-$strongly closed graph function then h has sem^*-closed [resp. $sem^{**}-$closed] graph function .

Proof : Let $z\in Z$ and each $r\in R$ such that $r\neq h(z)$. By hypothesis h has $\theta-$strongly closed graph function . So $\exists\ open\ K\subseteq Z\ containing\ z\ and\ \exists\ open\ L\subseteq R\ containing\ r$ S.t. $(\bar{K}\times L)\cap G(h)=\emptyset$. By proposition (2.8) we've got $K^{*z}\subseteq\bar{K}$. And by hypothesis R is *prefected* space so $L^{*r}\subseteq L$. we'll get $(K^{*z}\times L^{*r})\cap G(h)=\emptyset$. Thus h has sem^*-closed [resp. $sem^{**}-$closed] graph function .

Remark 3.31 : the following diagram holds .Let $h:(Z,\tau)\longrightarrow(R,\rho)$ then :



● Z is prefected space .

★ R is prefected space .

3.3 Diagram : The relationship between θ-strongly closed , strongly closed and new graph function .

Theorem 3.32 : The injective $A-$map function h from topological space (Z,τ) for I^*-T_2space (R,σ) . Then $G(h)$ is ω^*-closed graph , if (R,σ) is strongly .

Proof : Let $(z,r)\notin G(h)$, then $r\neq h(z)$ since R is I^*-T_2space so there exists subsets L_1 , L_2 in R such that $h(z)\notin L_2^{*r}$ and $r\notin L_1^{*h(z)}$ with $L_1^{*h(z)}\cap L_2^{*r}=\emptyset$. Since R is strongly so $h(z)\in L_1^{*h(z)}$ and $r\in L_2^{*r}$, with $L_1^{*h(z)}$ and L_2^{*r} are open subsets in R . But h is A-map, so $\exists\ K\subseteq Z$ containing z such that $h(K^{*z})\subseteq L_1^{*h(z)}$. Since $L_1^{*h(z)}\cap L_2^{*r}=\emptyset$ therefore $h(K^{*z})\cap L_2^{*r}=\emptyset$. Hence $(K^{*z}\times L_2^{*r})\cap G(h)=\emptyset$. Thus $G(h)$ is ω^*-closed graph .

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Theorem 3.33 : Let $h : (Z, \tau) \longrightarrow (R, \rho)$ is bijective continuous function , R is Prefected , Hausdorff space , then $G(h)$ is $\beta^{**}-$ closed graph .

Proof : Let $(z,r) \notin G(h)$, then $r \neq h(z)$ since R is T_2space so there exists open subsets L_1, L_2 in R such that $h(z) \in L_1$ and $r \in L_2$ with $L_1 \cap L_2 = \emptyset$. And $L_1 \cap {}^{*r}pr(L_2) = \emptyset$, because R is prefected . By hypothesis h is continuous function so $h^{-1}(L_1) = K$ is an open subset in Z . Therefore $(K \times {}^{*r}pr(L_2)) \cap G(h) = \emptyset$. Thus $G(h)$ is β^*-closed graph .

Theorem 3.34 : A Bijective function $h : (Z, \tau) \longrightarrow (R, \rho)$, Z has strongly space, then Z is Hausdorff space , if h has continuous and $G(h)$ is $\omega^* -$ closed .

Proof : Let $z_1 \neq z_2 \in Z$, then $h(z_1) \neq h(z_2)$ and $(z_1, h(z_2)) \notin G(h)$ since $G(h)$ is ω^*-closed so there exists subset K in Z containing z_1 and open subset L in R containing $h(z_2)$ S.t. $h(K^{*z_1}) \cap L = \emptyset$, since Z is strongly so K^{*z_1} is an open subset in Z and containing z_1 . Since h is continuous , so $h^{-1}(L)$ is an open subset containing z_2 in Z. Hence $K^{*z} \cap h^{-1}(L) = \emptyset$. Therefore Z is Hausdorff space.

Corollary 3.35 : A Bijective function f from a prefected and strongly space X into a topological space Y , then X is Hausdorff space , if f is continuous and $G(f)$ is Strongly closed graph .

Proof : By theorem (3.27) $G(f)$ is ω^*-closed . And by above theorem (3.34) X is Hausdorff space

Corollary 3.36 : A Bijective function $h : (Z, \tau) \longrightarrow (R, \rho)$, Z has strongly space, then Z is Hausdorff space , if h is continuous and $G(h)$ is $\theta -$ Strongly closed graph .

Proof : By Th. (3.28) $G(h)$ is $m^* -$ closed . And by theorem (3.34) Z is Hausdorff space .

Theorem 3.37 : A Bijective function $h : (Z, \tau) \longrightarrow (R, \rho)$, Z has strongly space, then R is Hausdorff space , if h has open and $G(h)$ is $\omega^* -$ closed .

Proof : Let $r_1 \neq r_2 \in R$, then there exists $z \in Z$ such that $h(z) = r_1$. and $(z, r_2) \notin G(h)$ since $G(h)$ is $\omega^* -$ closed so there exists subset K in Z containing x & open subset L in R contained r_2 S.t. $h(K^{*z}) \cap L = \emptyset$, since Z is strongly so $K^{*z} \subseteq Z$ open subset and $h(K^{*z})$ is an open subset containing r_1 in R because h is open mapping . Hence $h(K^{*z}) \cap L = \emptyset$. Therefore R is Hausdorff space .

Corollary 3.38 : A Bijective function $h : (Z, \tau) \longrightarrow (R, \rho)$, Z has prefected strongly , then R is Hausdorff space , if h is open and $G(h)$ is *Strongly closed* graph .

Proof : By Th. (3.27) $G(h)$ is ω^*-closed . And by above Th. (3.37) R is Hausdorff space .

Corollary 3.39 : A Bijective function f from a strongly space X into a topological space Y , then Y is Hausdorff space , if f is open and $G(f)$ is $\theta -$Strongly closed graph .

Proof : By theorem (3.28) $G(f)$ is ω^*-closed . And by theorem (3.37) Y is Hausdorff space .

Theorem 3.40 : A Bijective function $h : (Z, \tau) \longrightarrow (R, \rho)$, Z has strongly, then Z has Hausdorff space , if h has $I^{**} -$ map and $G(h)$ is $sem^* -$ closed [resp. $sem^{**} -$ closed] .

Proof : Let $z_1 \neq z_2 \in Z$, then $h(z_1) \neq h(z_2)$and $(z_1, h(z_2)) \notin G(h)$. Since $G(h)$ is $sem^* -$ closed So $\exists$ $K \subseteq Z$ *containing* z_1 *and* $\exists$ $L \subseteq R$ *containing* $h(z_2)$ S.t. $h(K^{*z_1}) \cap L^{*h(z_2)} = \emptyset$. But h is I^{**}-map we'll get $h^{-1}\left(L^{*h(z_2)}\right) = (h^{-1}(L))^{*z_2}$, so $K^{*z_1} \cap (f^{-1}(V))^{*x_2} = \emptyset$. But Z is a strongly space so K^{*z_1} and $(h^{-1}(L))^{*z_2}$ are open subsets in Z with $z_1 \in K^{*z_1}$ and $z_2 \in (h^{-1}(L))^{*z_2}$. Therefore Z is Hausdorff space.

Corollary 3.41 : A Bijective function h from a prefected and strongly space Z into a topological space R , we'll get Z is Hausdorff , if h has I^{**}-map and $G(h)$ is *Strongly closed* graph .

Proof : By Th. (3.29) $G(h)$ is sem^*-closed . And by above Th. (3.40) Z is Hausdorff space .

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Corollary 3.42 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, Z has strongly, R prefected , then Z has Hausdorff , if h has I^{**}-map and $G(h)$ is $\theta -$ Strongly closed graph .

Proof : By Th. (3.30) $G(h)$ is s^*-closed . And by Th. (3.40) Z has Hausdorff.

Theorem 3.43 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, R has strongly, then R is Hausdorff space , if h has I^*-map and $G(h)$ is $sem^* -$ closed [resp. $sem^{**} -$ closed] .

Proof : Let $r_1 \neq r_2 \in R$, then there exist $z \in Z$ such that $h(z) = r_1$, and $(z, r_2) \notin G(h)$ since $G(h)$ is sem^*-closed so there exists a subset K in Z containing z and there exists a subset L in R containing r_2 such that $h(K^{*z}) \cap L^{*r_2} = \emptyset$. Because h is I^*-map we'll get $h(K^{*z}) = (h(K))^{*h(z)}$, so $(h(K))^{*h(z)} \cap L^{*r_2} = \emptyset$. But R is a strongly space so $(h(K))^{*h(z)}$ and L^{*r_2} are open subsets in R with $r_1 \in (h(K))^{*h(z)}$ and $r_2 \in L^{*r_2}$. Therefore R is Hausdorff space .

Corollary 3.44 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, Z has prefected , R strongly, then R is Hausdorff space , if h has I^*-map and $G(h)$ is *Strongly closed* graph .

Proof : By Th. (3.29) $G(h)$ is sem^*-closed . And by above Th. (3.43) R is Hausdorff.

Corollary 3.45 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, R has prefected & strongly, then R is Hausdorff , if h has I^*-map and $G(h)$ is $\theta -$ Strongly closed graph .

Proof : By Th. (3.30) $G(h)$ is sem^*-closed . And by Th. (3.43) R is Hausdorff space .

Theorem 3.46 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, Z has prefected, then Z has $I^* - T_2$space , if $G(h)$ has β^*-closed [resp. β^{**}-closed] .

Proof : Let $z_1 \neq z_2 \in Z$, then $h(z_1) \neq h(z_2)$and $(z_1, h(z_2)) \notin G(h)$. Since $G(h)$ is β^{**}-closed So $\exists$ $open\ K \subseteq Z\ containing\ z_1\ and\ \exists\ open\ L \subseteq R\ containing\ h(z_2)$ S.t. $h(K) \cap {}^{*h(z_2)}pr(L) = \emptyset$. But Z has a perfected space , so $K = {}^{*z_1}pr(K)$ and $h^{-1}\left[{}^{*h(z_2)}pr(L)\right] = {}^{*z_2}pr(h^{-1}\left[{}^{*h(z_2)}pr(L)\right])$. Hence ${}^{*z_1}pr(K) \cap {}^{*z_2}pr(h^{-1}\left[{}^{*h(z_2)}pr(L)\right]) = \emptyset$ with $z_2 \notin {}^{*z_1}pr(K)$ and $z_1 \notin {}^{*z_2}pr(h^{-1}\left[{}^{*h(z_2)}pr(L)\right])$. Therefore Z is $I^* - T_2$space .

Corollary 3.47 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, Z has prefected , then Z is $I^* - T_2$space , if $G(h)$ is *Strongly closed graph.*

Proof : By Th. (3.25) $G(h)$ is β^*-closed . And by above Th. (3.46) Z has $I^* - T_2$space .

Corollary 3.48 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, Z , R are prefected , then Z is $I^* - T_2$space , if $G(h)$ has $\theta -$ Strongly closed graph .

Proof : By Th. (3.26) $G(h)$ is β^*-closed . And by Th. (3.46) Z has $I^* - T_2$space .

Theorem 3.49 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, then Z is $I^* - T_2$space , if h is I^{**}-map and $G(h)$ has sem^*-closed [resp. sem^{**}-closed] *graph* function .

Proof : Let $z_1 \neq z_2 \in Z$, then $h(z_1) \neq h(z_2)$and $(z_1, h(z_2)) \notin G(h)$. Since $G(h)$ is sem^*-closed So $\exists$ $K \subseteq Z\ containing\ z_1\ and\ \exists\ L \subseteq R\ containing\ h(z_2)$ S.t. $h(K^{*z_1}) \cap L^{*h(z_2)} = \emptyset$. Since h is I^{**}-map so we get that $h^{-1}\left(L^{*h(z_2)}\right) = (h^{-1}(L))^{*z_2}$, which implies that $K^{*z_1} \cap (h^{-1}(L))^{*z_2} = \emptyset$, with $z_1 \notin K^{*z_1}$ and $z_2 \notin (h^{-1}(L))^{*z_2}$. Therefore Z is $I^* - T_2$space .

Corollary 3.50 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, Z has prefected , then Z is $I^* - T_2$space , if h has I^{**}-map and $G(h)$ is *Strongly closed graph.*

Proof : By Th. (3.29) $G(h)$ is sem^*-closed . And by above Th. (3.49) Z is $I^* - T_2$space .

Corollary 3.51 : A Bijective function $h : (Z,\tau) \longrightarrow (R,\rho)$, R has prefected , then Z is $I^* - T_2$space , if h has I^{**}-map and $G(h)$ is $\theta -$ Strongly closed graph .

Proof : By Th. (3.30) $G(h)$ is sem^*-closed . And by Th. (3.49) Z is $I^* - T_2$space .

Theorem 3.52 : A Bijective function $h\ :\ (Z\ ,\tau)\ \longrightarrow\ (\ R\ ,\rho\ )$, R has prefected , then R is $I^* - T_2$space , if $G(h)$ is β^*-closed [resp. β^{**}-closed] .

Proof : Let $r_1 \neq r_2 \in R$, then $\exists z \in Z$ such that $h(z) = r_1$ and $(z, r_2) \notin G(h)$. Since $G(h)$ is β^*-closed $\exists$ $open\ K \subseteq Z\ containing\ z\ and \ \exists\ open\ L \subseteq R\ containing\ r_2$ S.t. $h(K) \cap {}^{*r_2}pr(L) = \emptyset$. But R is a perfected space , so $h(K) = {}^{*r_1}pr(h(K))$. Hence ${}^{*r_1}pr(h(K)) \cap {}^{*r_2}pr(L) = \emptyset$ with $r_1 \notin {}^{*r_2}pr(L)$ and $r_2 \notin {}^{*r_1}pr(h(K))$. Therefore R is $I^* - T_2$space .

Corollary 3.53 : A Bijective function $h\ :\ (Z\ ,\tau)\ \longrightarrow\ (\ R\ ,\rho\ )$, R has prefected , then R is $I^* - T_2$space , if $G(h)$ is *Strongly closed graph* [θ *−Strongly closed graph*] .

Proof : By Th. (3.25)[(3.26)] $G(h)$ is β^*-closed . And by above Th. (3.52) R is $I^* - T_2$space .

Theorem 3.54 : A Bijective function $h\ :\ (Z\ ,\tau)\ \longrightarrow\ (\ R\ ,\rho\ )$, Z has prefected , then R is $I^* - T_2$space , if h is I^*-map and $G(h)$ is $\beta^* -$ closed [resp. $\beta^{**} -$ closed] .

Proof : Let $r_1 \neq r_2 \in R$, then $\exists\ z \in Z$ such that $h(z) = r_1$. Since $G(h)$ is β^*-closed so $\exists$ $open\ K \subseteq Z\ containing\ z\ and\ \exists\ open\ L \subseteq R\ containing\ r_2$ S.t. $h(K) \cap {}^{*r_2}pr(L) = \emptyset$. But R is a perfected space and h is I^*-map then by Th. (2.1.14) , we have $h(K) = {}^{*r_1}pr(h(K))$. Hence ${}^{*r_1}pr(h(K)) \cap {}^{*r_2}pr(L) = \emptyset$ with $r_1 \notin {}^{*r_2}pr(L)$ and $r_2 \notin {}^{*r_1}pr(h(K))$. Therefore R is $I^* - T_2$space .

Corollary 3.55 : A Bijective function $h\ :\ (Z\ ,\tau)\ \longrightarrow\ (\ R\ ,\rho\ )$, Z has prefected , then $R$is $I^* - T_2$space , if h is I^*-map and $G(h)$ is Strongly closed graph .

Proof : By Th. (3.25) $G(h)$ is β^*-closed . And by above Th. (3.54) R is $I^* - T_2$space .

Corollary 3.56 : A Bijective function $h\ :\ (Z\ ,\tau)\ \longrightarrow\ (\ R\ ,\rho\ )$, Z ; R have prefected , then R is $I^* - T_2$space , if fh is I^*-map and $G(h)$ is θ −Strongly closed graph .

Proof : By Th. (3.26) $G(h)$ is β^*-closed . And by above Th. (3.54) Y is $I^* - T_2$space .

Theorem 3.57 : A Bijective function $h\ :\ (Z\ ,\tau)\ \longrightarrow\ (\ R\ ,\rho\ )$, then R is $I^* - T_2$space , if h is I^*-map and $G(h)$ is sem^*-closed [resp. sem^{**}-closed] *graph function* .

Proof : Let $r_1 \neq r_2 \in R$, then $\exists\ z \in Z$ such that $h(z) = r_1$and $(z, r_2) \notin G(h)$. Since $G(h)$ is sem^* -closed $\exists$ $K \subseteq Z\ containing\ z\ and\ \exists\ L \subseteq R\ containing\ r_2$ S.t. $h(K^{*z}) \cap L^{*r_2} = \emptyset$. Since h is I^*-map so we get that $h(K^{*z}) = (h(K))^{*r_1}$, which implies that $(h(K))^{*r_1} \cap L^{*r_2} = \emptyset$, with $r_1 \notin L^{*r_2}$ and $r_2 \notin (h(K))^{*r_1}$. Therefore R is $I^* - T_2$space .

Corollary 3.58 : A Bijective function $h\ :\ (Z\ ,\tau)\ \longrightarrow\ (\ R\ ,\rho\ )$, Z has prefected , then R is $I^* - T_2$space , if h is I^*-map and $G(h)$ is *Strongly closed graph* .

Proof : By Th. (3.29) $G(h)$ is sem^*-closed . And by above Th. (3.57) R is $I^* - T_2$space .

Corollary 3.59 : A Bijective function $h\ :\ (Z\ ,\tau)\ \longrightarrow\ (\ R\ ,\rho\ )$, R has prefected , then R is $I^* - T_2$space , if h is I^*-map and $G(h)$ is θ *−Strongly closed graph* .

Proof : By Th. (3.30) $G(h)$ is sem^*-closed . And by above Th. (3.57) R is $I^* - T_2$space .

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