International Journal of Scientific Trends- (IJST)
ISSN: 2980-4299
Volume 4, Issue 9, September - 2025
Website: https://scientifictrends.org/index.php/ijst
Open Access, Peer Reviewed, Scientific Journal

Examining Role of Mathematics in Hardware Engineering: Relationship Between Linear Algebra and Embedded Systems

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Abstract

Linear algebra is often regarded as an abstract branch of mathematics, however, it has a myriad of implementations in the real world, particularly in the field of computer science and hardware engineering. Embedded systems, which represent small computing units integrated into various devices that we use on a daily basis, heavily rely on mathematical models in order to function properly. This article examines the role that fundamental linear algebra concepts such as vectors, matrices, eigenvalues and linear transformations play in embedded systems. Particularly, it outlines practical implementation of these concepts in signal processing, control systems, robotics and image processing, highlighting how they support core tasks and enable efficient performance. By bridging mathematical theory with engineering practice, this paper shows why linear algebra is indispensable for modern embedded system design and development.

Keywords:

Introduction

In today's cutting-edge era, embedded systems work as a backbone of numerous devices that have become widespread and are constantly used in our daily lives. Everything from smartphones and household appliances to medical equipment and industrial machines function using small computing units that help to perform specific tasks. They expand human capability to digitize many fields, thereby facilitating their own labor and bringing our lives to the next level of comfort. However, the rise of such sophisticated systems requires robust mathematical models that can ensure accuracy and efficiency in how these systems operate.

The vast majority of students who pursue a degree in computer science or hardware engineering often question whether theoretical subjects in math such as linear algebra will ever be useful for them. However, it is linear algebra that proves to be one of the most effective mathematical tools for working with embedded systems. It is true that concepts such as vectors, matrices, eigenvalues, and linear transformations may at first appear abstract in traditional mathematics courses, yet, in the real world they serve as a foundation for a practical implementation of signal processing, robotics, control systems, and machine learning.

This article aims to demonstrate the connection between theory and practice, by examining how linear algebra is applied within embedded systems. Firstly, the article will give a brief explanation of basic concepts of linear algebra, followed by an overview of the architecture of embedded systems. Thus, the paper refutes the common myth that theoretical knowledge in mathematics is rarely put into practice, by showing how crucial linear algebra is implementing engineering practices.

1. Fundamental concepts of linear algebra

Linear algebra is a branch of mathematics that deals with abstract concepts. In contrast with calculus which deals with derivatives and integrals, and focuses on continuous change and accumulation, linear algebra demonstrates relationship between variables that are linear. Majority of people tend to believe only calculus is important in engineering, however linear algebra provides a way to compute multidimensional data effectively, hence, playing a key role in handling computations.

Vectors and matrices. In mathematics and physics, vectors are defined as quantities that capture a magnitude and a direction (e.g., a distance vector). Matrices, in mathematics, are meant to represent linear mappings, which is defined as the mapping between two vector spaces that preserve vector addition and scalar multiplication. Both vectors and matrices are used in data representation. When working with data, we often want to manipulate them and / or feed them into machine learning models. This process involves a lot of computation and will often require adding and multiplying many numbers, and for this reason vectors and matrices are used - they provide better computational efficiency [1]. Besides that, vectors and matrices are used in signal processing. They enable complex operations like filtering, compression and transformation of signals. Basic matrix operations and decomposition techniques form the foundation for solving signal processing problems and perform tasks like noise reduction and data compression [2].

Eigenvalues and eigenvectors. Eigenvalues and eigenvectors are a special set of scalars and vectors respectively, associated with a linear system of equations (i.e., a matrix equation), always paired together. The determination of the eigenvalues and eigenvectors of a system is extremely important in physics and engineering, where it is equivalent to matrix diagonalization and is commonly applied in stability analysis, the physics of rotating bodies, and vibration systems. [3]. Eigenvalues and eigenvectors are used in Principal Component Analysis (PCA). Eigenvectors of the Covariance matrix (used to identify correlation between variables) are the directions of the axes where there is most variance (most information) and that are called Principal Components. And eigenvalues are the coefficients attached to eigenvectors, which give the amount of variance carried in each Principal Component. By ranking eigenvectors in order of their eigenvalues, highest to lowest, the principal components in order of significance is obtained [4]. PCA, in its turn, is directly tied to robotics applications.

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Linear transformations. In mathematics, a linear transformation is a mapping between vector spaces that preserves vector addition and scalar multiplication, and is typically represented by a matrix. In data science and machine learning, the term is used more broadly: matrix multiplications in neural network layers, projections in PCA, and scaling or rotating data are all examples of linear transformations. While not always tied to the strict mathematical definition, the idea remains the same: linear transformations provide structured, predictable ways to manipulate and represent data [5].

Thus, it can be seen that linear algebra concepts are used in various fields- from data representation and signal processing to robotics, and form a ground for many algorithms in embedded systems as well.

3. Application in Embedded Systems

Embedded systems are basically computers built into other devices to do specific jobs. They use a tiny brain, like a microprocessor or microcontroller, and they're everywhere - from simple things like digital watches to more complex stuff like hybrid vehicles and even the technology used in planes. Embedded systems are built to do specific jobs within bigger machines or electrical setups. They're made up of a bunch of processors, circuit boards, and software that all work together to get certain tasks done [6]. The core of any embedded system is either microcontroller or microprocessor- which operates as the system's brain, handles calculations and processes data. Also there are memory chips for keeping instructions and data, and finally, peripheral devices for input/output.

3.1 Signal processing

Signal processing plays a crucial role in modern-day embedded systems. It mainly works with manipulating digital and analog signals to extract data or convert them into different form. Those processed signals can originate from sensors, transducers etc. As for the math used behind signal processing – convolution is used in digital signal processing to study and design linear time-invariant (LTI) systems such as digital filters. The output signal, $y[n]$, in LTI systems is the convolution of the input signal, $x[n]$ and impulse response $h[n]$ of the system (Fig 1). The convolution theorem is used to design filters in the frequency domain [7].

Another mathematical concept used is matrices. They are implemented in digital filters, since every linear digital filter can be expressed as a constant matrix **h** multiplying the input signal x to produce output signal(vector) y, resulting in: $y=\mathbf{h}x$. Example is the general linear, time-invariant (LTI) matrix called Toeplitz. A Toeplitz matrix is constant along all its diagonals. For example, the general LTI matrix is given by Fig 2.1. Restricting it to causal LTI filters (whose output does not depend on any future inputs) gives Fig 2.2 [8]

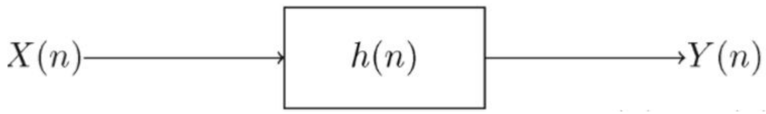

Fig 1. Convolution for linear time-invariant systems.

$$\mathbf{h} = \begin{bmatrix} h_0 & h_{-1} & h_{-2} \\ h_1 & h_0 & h_{-1} \\ h_2 & h_1 & h_0 \end{bmatrix} \mathbf{h} = \begin{bmatrix} h_0 & 0 & 0 \\ h_1 & h_0 & 0 \\ h_2 & h_1 & h_0 \end{bmatrix}.$$

Fig 2.1 Fig 2.2

General LTI matrix

3.2 Control Systems

Embedded control systems often rely on state-space models, which provide a mathematical framework for analyzing and designing control systems in the time domain. They represent the system dynamics using a set of first-order differential equations. State-space equations describe the evolution of state variables (set of variables, that completely describe the dynamic behavior of a system at any given time) and the relationships between state variables, inputs and outputs. Input equation is: x'(t)= Ax(t)+Bu(t), where A is a system matrix, B input matrix, state vector x(t), and input vector u(t). The output equation is: y(t)=Cx(t)+Du(t), where y(t) is output vector, C is output matrix and D is a feedthrough matrix. State-space models facilitate the design of state feedback controllers, where the control input is determined based on measured or estimated state variables [9]. Importantly, the eigenvalues of A determine stability: for continuous-time systems all eigenvalues must have negative real parts; for discrete-time systems (common in digital controllers) all eigenvalues must lie inside the unit circle.

As a result of progression of embedded control systems, modern embedded platforms enable complex control schemes like Field-Oriented Control (FOC). FOC allows embedded systems to deliver smooth, efficient torque control in motors—critical in drones, EVs and robotics. Without embedded systems FOC wouldn't exist, since it requires real-time digital signal processing and advanced control algorithms (execution of Space Vector PWM in real-time) [10].

3.3 Robotics

Robots are the greatest representation of implementation of linear algebra. Without it, robots would not be able to move with precision, orient themselves in space, or interact effectively with their environment. Since robots do not simply move in one direction, but operate in complex, multi-dimensional environment, linear algebra provides a framework that handles such problems. Robot's position in space can be described by a vector. A planar(2D) robot can be represented by a vector like **[x, y]**, while a robot in three-dimensional space relies on **[x, y, z]**. These vectors form the foundation of robotic kinematics. Besides that, matrices manipulate those vectors and work as operators that tell robots how to move. Rotation matrices allow precise orientation changes, while scaling matrices control how movements expand or contract. Also there are translation matrices that are responsible for the shift from one point in space to another. Robotic kinematics often work with **homogeneous coordinates**. By extending vectors and matrices into four dimensions, rotation, translation, and scaling can be combined into a single matrix multiplication- and for this transformation matrix is used (Fig 3). In transformation matrix R is 3x3 rotation matrix and t is a translation vector.

Moreover, linear algebra used not only in robot kinematics, but in robot vision as well. Computer vision systems use matrix operations for image transformations, object recognition, and depth estimation. Cameras capture light in two dimensions, but robots must interpret this into 3D understanding. Matrices handle the conversion between coordinate systems of camera and world. Hence, whole field of robotics heavily relies on linear algebra concepts [11].

$$T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

Fig 3. Transformation matrix

3.4 Image Processing

Majority of modern embedded systems such as smartphones, drones and autonomous vehicles often include cameras. This creates the need for algorithms that can process captured images and translate raw pixel data into actual information. Since all images are represented as matrices of pixel values, linear algebra becomes a tool that helps to perform different operations on those matrices.

Section 3.1 discussed about convolution and how it used in digital signal processing. In the same way convolutional filtering can be used in image processing for implementing algorithms like edge detection, image sharpening and image blurring . This is done by selecting the appropriate convolution matrix (kernel) , and by performing a convolution between the kernel and the image [12].

As for the computer vision, it is obvious that image processing plays a key role. Implementation of the core tasks such as flipping images, rotation, noise reduction, compressing are performed using matrices. For example, for implementing flip or flop or rotation operations the transpose operation of matrices is suitable .

In the section 2, when talking about eigenvalues and eigenvectors, it was noted how PCA uses them . Principal Component Analysis is a powerful tool , which is mathematically a computation of eigenvectors, that helps to reduce the dimensions of data and discard the unnecessary dimensions, keeping only needed features. PCA uses Eigenface method (method which project linearly the image space to a low dimensional image feature space) in order to perform reduction of dimensions. This method is used in object detection and facial recognition [13].

4. Conclusion

This article has examined how linear algebra, which is often treated with skepticism among university students, directly supports the proper functioning of many modern embedded systems. Concepts such as vectors, matrices, eigenvalues and linear transformations create a mathematical framework for solving engineering problems which occur within the field of embedded systems. Linear algebra concepts are involved in signal processing by using matrices, in control systems state-space models are used, robotic kinematics and vision are implemented using matrix manipulations and image processing uses PCA which is computation of eigenvalues.

This demonstrates that behind all the devices used on a daily basis there stands a practical implementation of mathematics. Mathematical knowledge is not restricted to a mere tedious theory, which at first glance seems disconnected from real-world technology, but rather, math is a dialect which translates engineering ideas into real actions.

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