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The Earth Potential Perturbation on Low Earth Satellites Orbits at Different Inclinations

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Abstract

It is helpful to illustrate the potential using changes in the orbit components, {semi major axis (a) eccentricity (e) inclination (i) , right ascension of ascending nodes (Ω)}, These components don't change when there are no disturbances. In this paper we analyze how the earth could disturb low-earth orbit satellites for both prograde and retrograde orbits. The program celestial mechanics was used to produce the results for the period one day, which corresponds to around 16 revolutions. The findings demonstrate that the inclination of the satellites affects how the earth's potential disturbance affects the orbital elements of low earth orbiting satellites, and that the effects of perturbations on prograde orbits are opposite to those caused on retrograde orbits.

The magnitude of the eccentricity perturbations fluctuates inversely with changing inclination for prograde orbits and directly with inclination for retrograde orbits, with such a period equal to the period of the satellite's revolution. Also we conclude that for prograde orbits, the largest departure from the initial osculating Keplerian orbit happens at small inclinations along the track and in the radial direction, whereas the out of plane direction shows little variation at large inclinations.

Keywords: Perturbations, Orbital elements, Gravitational field, Prograde, Retrograde.

Introduction

The unperturbed motion in celestial mechanics is the orbit motion of two spherically symmetric bodies as stated by the equations of motion [1]. Perturbations are changes out of a typical, idealized, or unperturbed motion. One of most suitable technique for detecting perturbations is numeric analysis. Examining the effects of perturbations can be done using one of three primary methods: special perturbation techniques, general perturbation techniques or semi analytical techniques. All of the forces that affect the satellite's orbit, along with the accelerations generated by the central body, drag, the third body, the pressure of solar radiation, and many others, are recognized as perturbation forces, even though the majority of them are very small and are usually overlooked. As computational resources have increased, it has become possible to study tide effects as a perturbation force [2].

Only the main relationships are mentioned in the analysis. In addition to Mueller (1964) [3], Groten (1979, 1980) [4], and Kaula (1966) [5], detailed derivations and debates can be found in these books. Khan provides a concise and understandable summary (1983) [6]. In particular, Kaula (1966) [5].

In the past few generations, many researchers are interested in researching the satellite's motion and its lifetime. To solve perturbed equations of motion, various analytical, semi-analytic, and numerical techniques are applied.

A model for the orbit propagator for LEO satellites that contains dominant perturbations was created by Asma [7]. In order to optimize the propagation, this simulation models the perturbing forces. S. Mishra etal [8]. The behavior of orbital elements was thoroughly examined by Mohammed and Abdul-Rahman using various A/M, altitudes, and eccentricities [9].

Theory of Perturbation

To gain more in-depth analyses of the relationship between Earth's anomalous gravitational potential and perturbations of the satellite orbit, we can use a series expansion with spherical harmonics for the potential [10].

$$V = \frac{GM}{r}\left(1 + \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a_e}{r}\right)^n (C_{nm} \cos\cos m\lambda + S_{nm} \sin\sin m\lambda) P_{nm}(\cos\cos\vartheta)\right)$$

The equatorial radius is represented by $\boldsymbol{a_e}$, and $\boldsymbol{P_{nm}}$ represent Legendre functions or Legendre polynomials. The harmonic coefficients $\boldsymbol{C_{nm}}$ and $\boldsymbol{S_{nm}}$ are integrals of the mass and characterize the distribution of mass within the main body [11].

First component, GM/r, is also known as the Keplerian term since it pertains to Keplerian motion and explains the potential of a homogeneous sphere. The remaining double-summation expressions are caused by the unsettling potential R. The phrases in n = 1 and m = 0,1 become zero as origin of the coordinate system is moved to the primary's center of mass.

$$R = \frac{GM}{r}\left(\sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{a_e}{r}\right)^n (C_{nm} \cos\cos m\lambda + S_{nm} \sin\sin m\lambda) P_{nm}(\cos\cos\vartheta)\right)$$

These coefficients are given names.

zonal m = 0,

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tesseral m ≠ 0,
sectorial m = n.

The disturbing potential was reformulated into the perturbation equations as below [3, 5, and 6].

$$\frac{d\Omega_{nmpq}}{dt} = \frac{GMa_e^n F'_{nmp} G_{npq} S_{nmpq}}{\underline{n}a^{n+3}\sqrt{1-e^2}\, sin\, sin\, i},$$

$$\frac{di_{nmpq}}{dt} = \frac{GMa_e^n F'_{nmp} G_{npq} S'_{nmpq}}{\underline{n}a^{n+3}\sqrt{1-e^2}\, sin\, sin\, i}\left((n-2p)\, cos\, cos\, i\, - m\right),$$

$$\frac{d\omega_{nmpq}}{dt} = GMa_e^n\left(\frac{\sqrt{1-e^2}}{e}F_{nmp}G'_{npq} - \frac{cot\, cot\, i}{\sqrt{1-e^2}}F'_{nmp}G_{npq}\right)\frac{S_{nmpq}}{\underline{n}a^{n+3}}$$

$$\frac{da_{nmpq}}{dt} = \frac{2GMa_e^n F_{nmp} G_{npq} S'_{nmpq}}{\underline{n}a^{n+2}}(n-2p+q),$$

$$\frac{de_{nmpq}}{dt} = \frac{GMa_e^n F_{nmp} G_{npq} S'_{nmpq}}{\underline{n}a^{n+3}e}\left((1-e^2)(n-2p+q) - \sqrt{1-e^2}(n-2p)\right),$$

$$\frac{d\underline{M}_{nmpq}}{dt} = \frac{GMa_e^n F_{nmp} S_{nmpq}}{\underline{n}a^{n+3}}\left(2(n+1)G_{npq} - \frac{1-e^2}{e}G'_{npq}\right) + \underline{n}\,.$$

The functions S', F', and G' are derivatives of the relevant arguments.

It is convenient to separate the perturbations into three groups according to their periods using the explicit algebraic formulation. These long- and short-term perturbations are secular (linear). You can overlay them Figure (1).

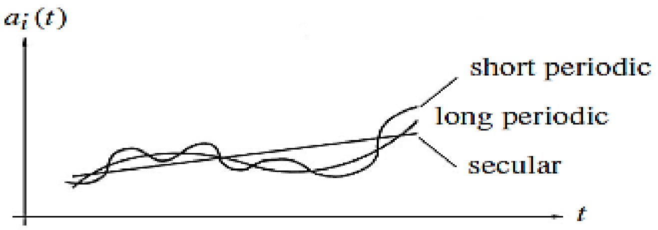

Figure 1. Perturbations of the elements $a_i(t)$ (t) [12].

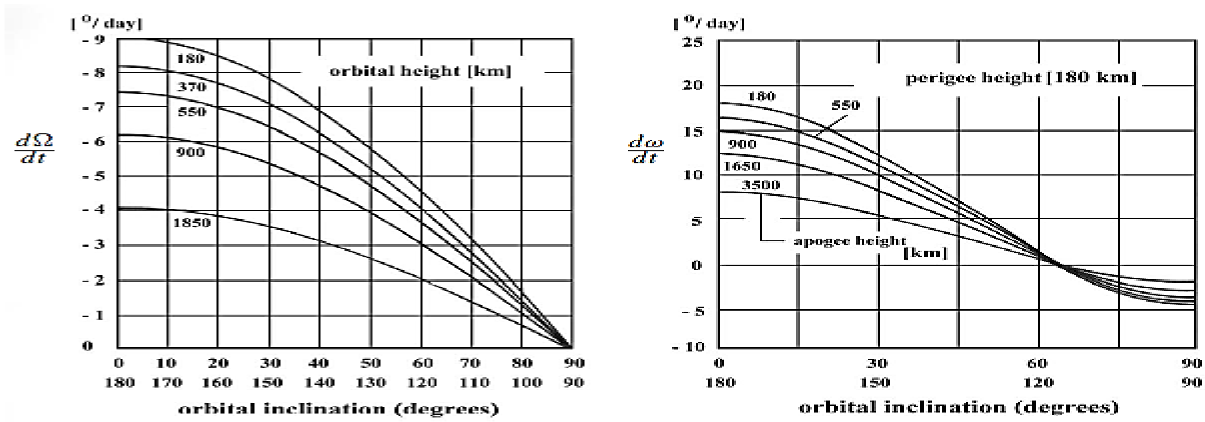

Figure 2. Shows the relationship between inclination, and right ascension (left); and the relationship between inclination and argument of perigee (right) (right) [13].

It is possible to give the perturbation equations an elegant form.

$$\dot{I}_k = \nabla_v I_k \cdot \nabla_r R \,, \qquad k = 1,2,\dots,6.$$

In terms of the vectors h and q, the perturbation equation of classical osculating elements are as follows: [1]

$$\dot{p} = \frac{2}{\mu} h \cdot \dot{h}$$

$$\frac{di}{dt} = -\frac{1}{\sqrt{h_1^2 + h_2^2}} \left\{ \dot{h} \cdot e_3 - \frac{h_3}{h} \dot{h} \right\}$$

$$\dot{\Omega} = \frac{1}{h_1^2 + h_2^2} \left\{ \left(h \times \dot{h} \right) \cdot e_3 \right\}$$

$$\dot{e} = \frac{1}{\mu q} q \cdot \dot{q}$$

$$\dot{\omega} = \frac{1}{q \sin \sin \omega} \{ \cos \cos \omega \, \dot{q} - e_\Omega \cdot \dot{q} \} - \cos \cos i \, \dot{\Omega}$$

$$\dot{T}_0 = -\frac{3}{2a}(t - T_0)\dot{a} + \frac{\sqrt{|1 - e^2|}}{en \ \sin \sin v} \cdot \{ \left[- \cos \cos v \, \dot{e} + \frac{r}{a^2} \, \dot{a} \right] , \qquad e < 1 \qquad \left[+ \cos \cos v \dot{e} + \frac{r}{a^2} \, \dot{a} \right] , \qquad e > 1$$

The elements p, i, Ω, e, ω and T_0 are represented by a self-contained differential equation system formed by the aforementioned equations.

Results and Discussion

For both prograde and retrograde orbits, we analyze in this paper how the earth could disturb low-earth orbit satellites.

The program celestial mechanics was used to produce the results for the period one day, which corresponds to around 16 revolutions.

Half of the satellite's orbital cycle around the earth corresponds to the primary period of the perturbations in semimajer axis (a). Variation in amplitude is changed by changing the inclination; for prograde orbits, it increases with increasing inclination, while for retrograde orbits, it decreases with increasing inclination.

The amplitude of the eccentricity perturbations varies inversely with changing inclination for prograde orbits and directly with inclination for retrograde orbits, with the period being equivalent to the rotational period of the satellite.

The inclination of the satellites has a significant impact on the growth of the mean right ascension of the ascending node (Ω) , as the Figures (3 - 8) demonstrate for various values of inclination (i).

And we observe that the values of (Ω) change inversely as inclination grows for prograde orbits, whereas the values of (Ω) increase as inclination values decrease for retrograde orbits.

For prograde orbits with small inclination and for retrograde orbits with large inclination, the node's value of regression is maximum.

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The Figures(10) demonstrate that for prograde orbits, the biggest deviation from the initial osculating Keplerian orbit occurs at small inclinations along the track and in the radial direction, whereas the out of plane direction exhibits little variation at large inclinations.

Conclusion

The results lead us to the conclusion that the inclination of the satellites affects how the earth's potential disturbance affects the orbit elements for low earth orbiting satellite, and that the effects of perturbations on prograde orbits are opposite to those caused on retrograde orbits.

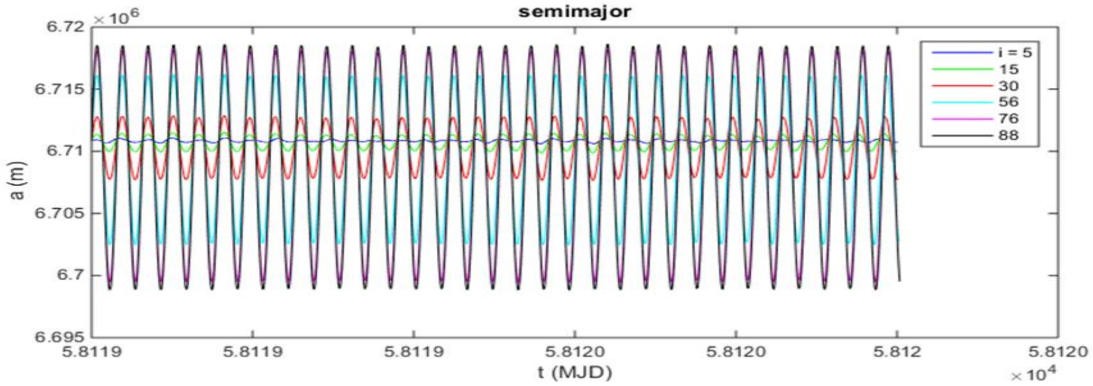

Figure 3. semimajor axis of a low orbits satellites in Earth's oblate gravitational field.

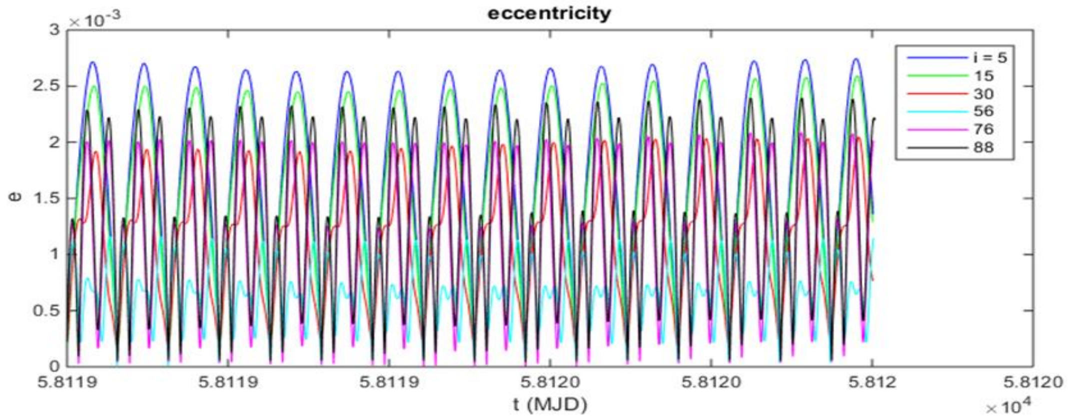

Figure 4. eccentricity of a low orbits satellites in Earth's oblate gravitational field.

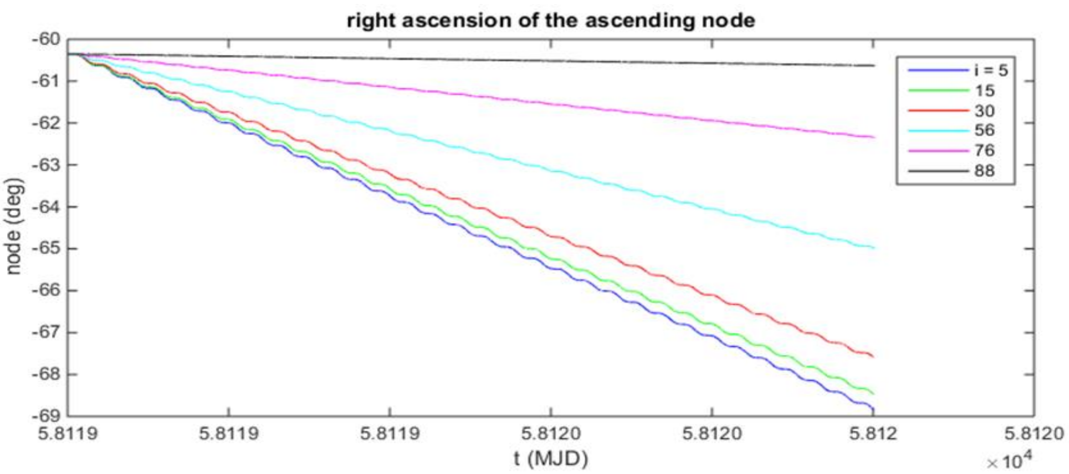

Figure 5. right ascension of a low orbits satellite in Earth's oblate gravitational field.

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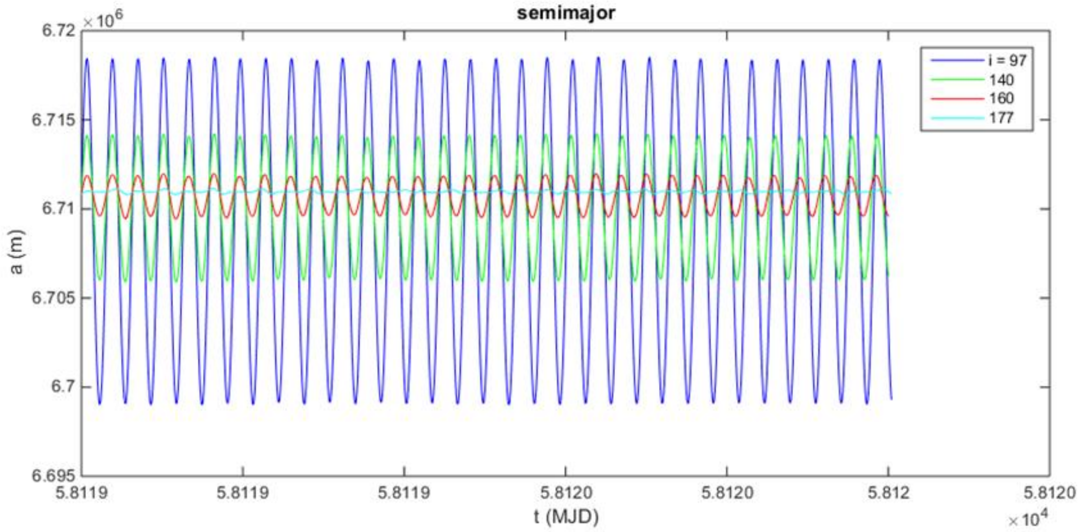

Figure 6. semimajor axis of a low orbits satellites in Earth's oblate gravitational field.

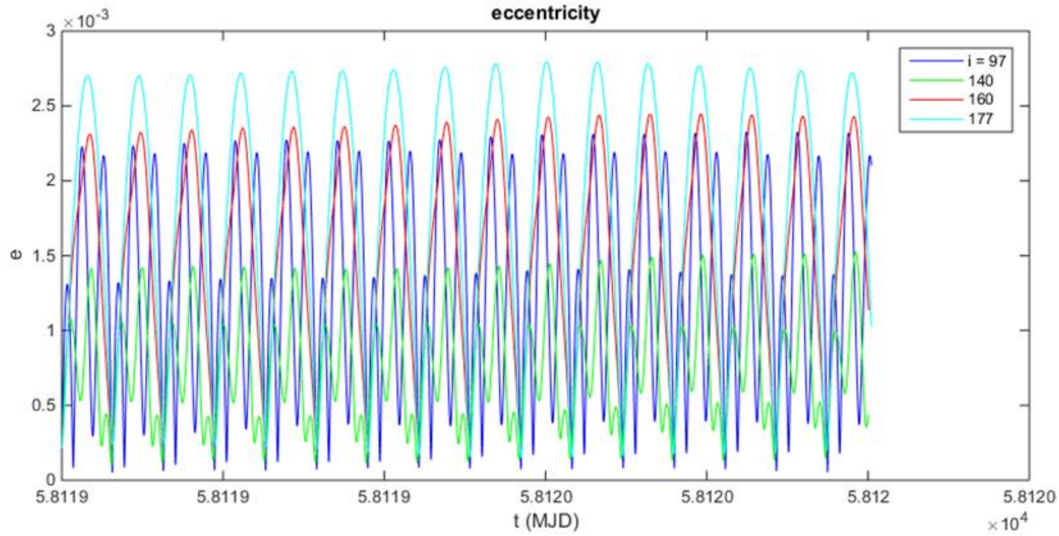

Figure 7. eccentricity of a low orbits satellites in Earth's oblate gravitational field.

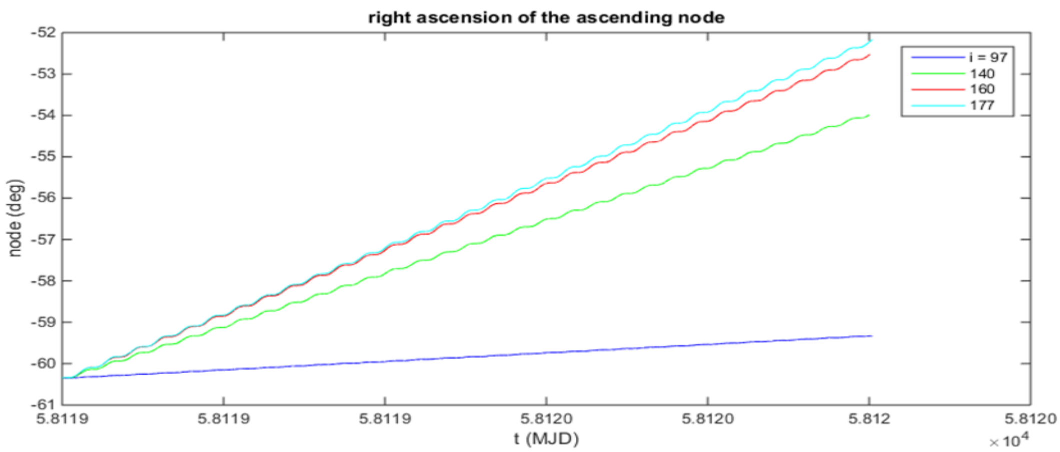

Figure 8. right ascension of a low orbits satellite in Earth's oblate gravitational field.

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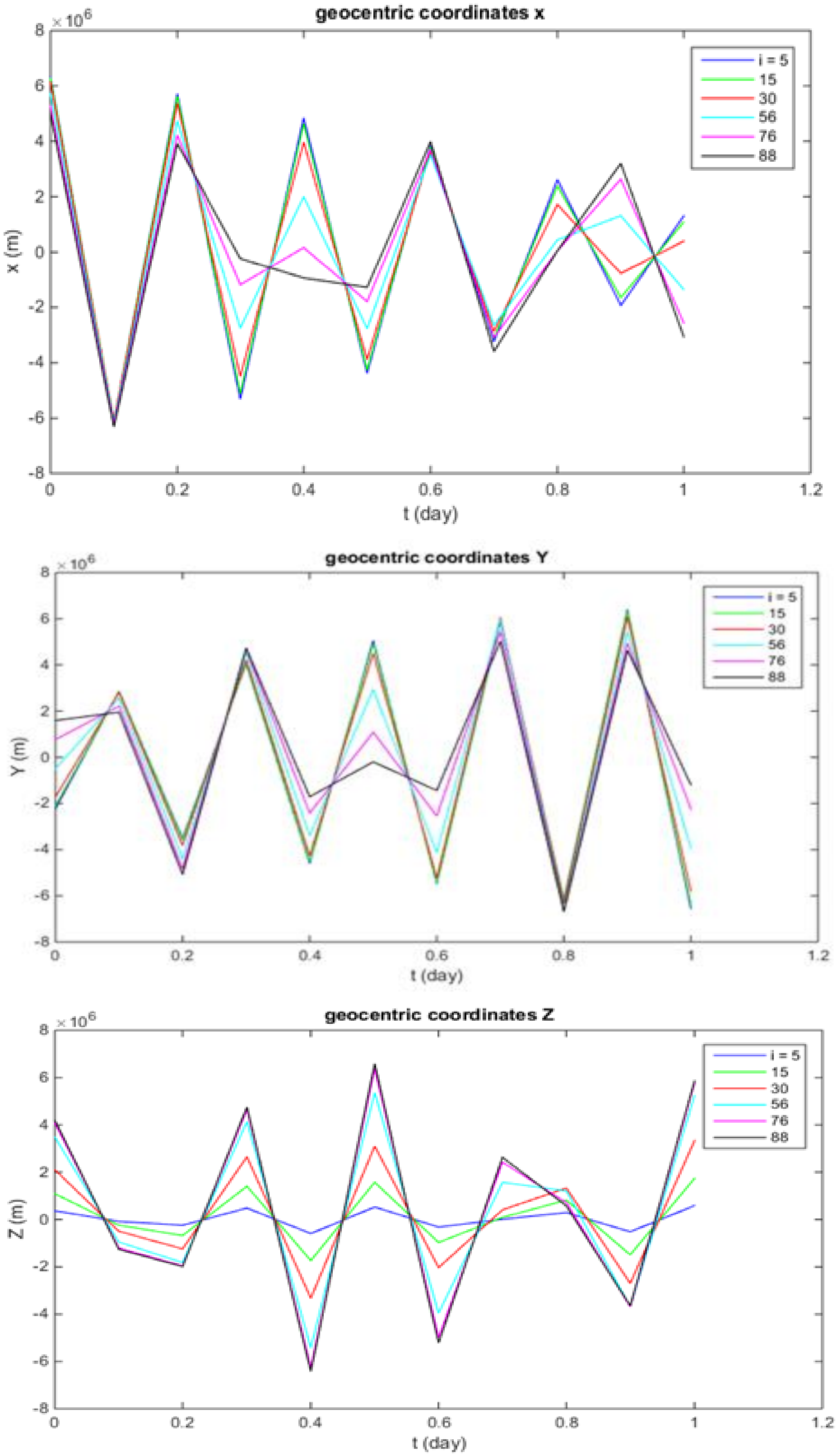

Figure 9. geocentric coordinates of a low orbits satellites in Earth's oblate gravitational field.

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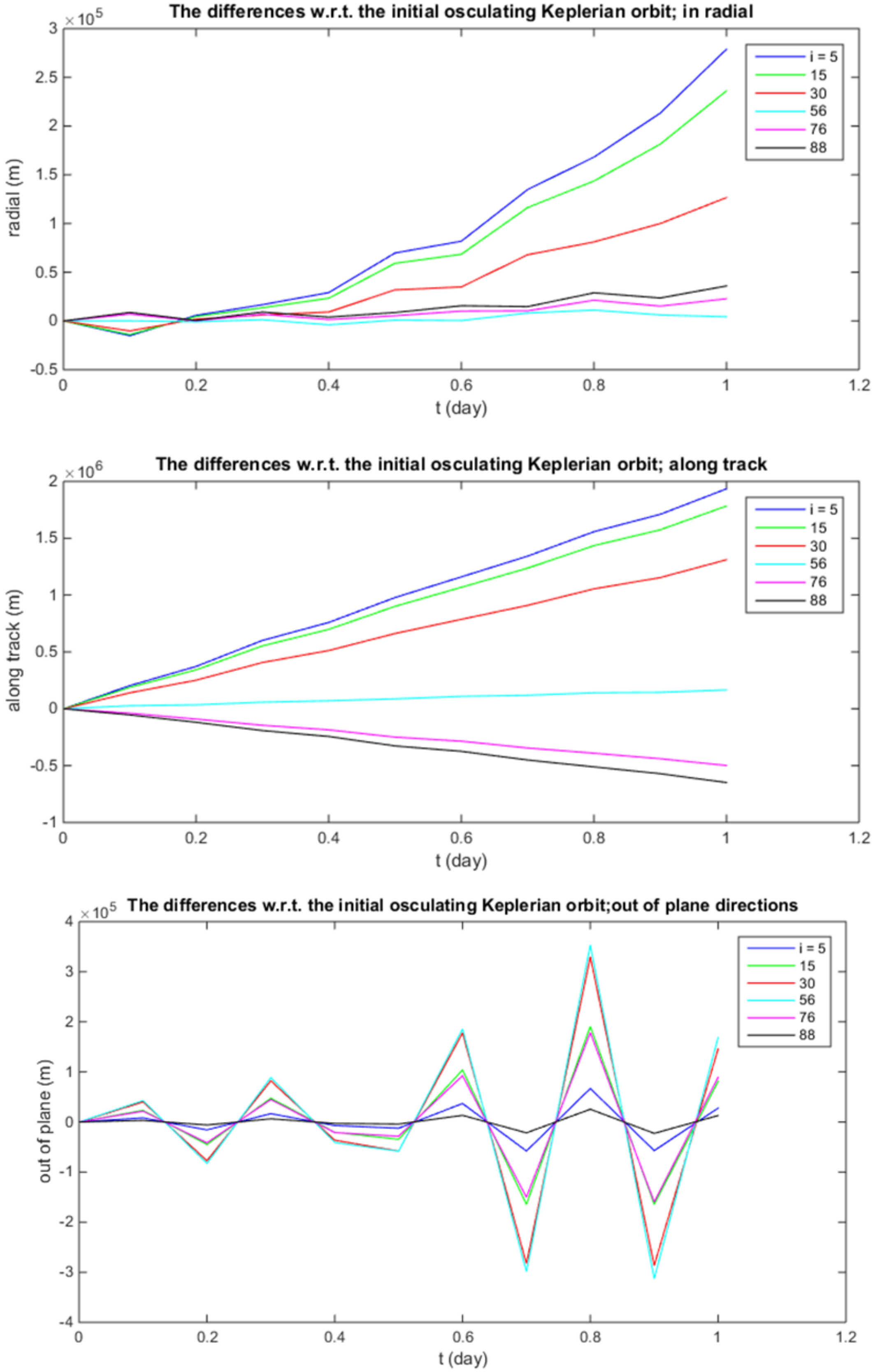

Figure 10. the differences w.r.t the initial osculating Keplerian orbit of a low orbits satellites in Earth's oblate gravitational field.

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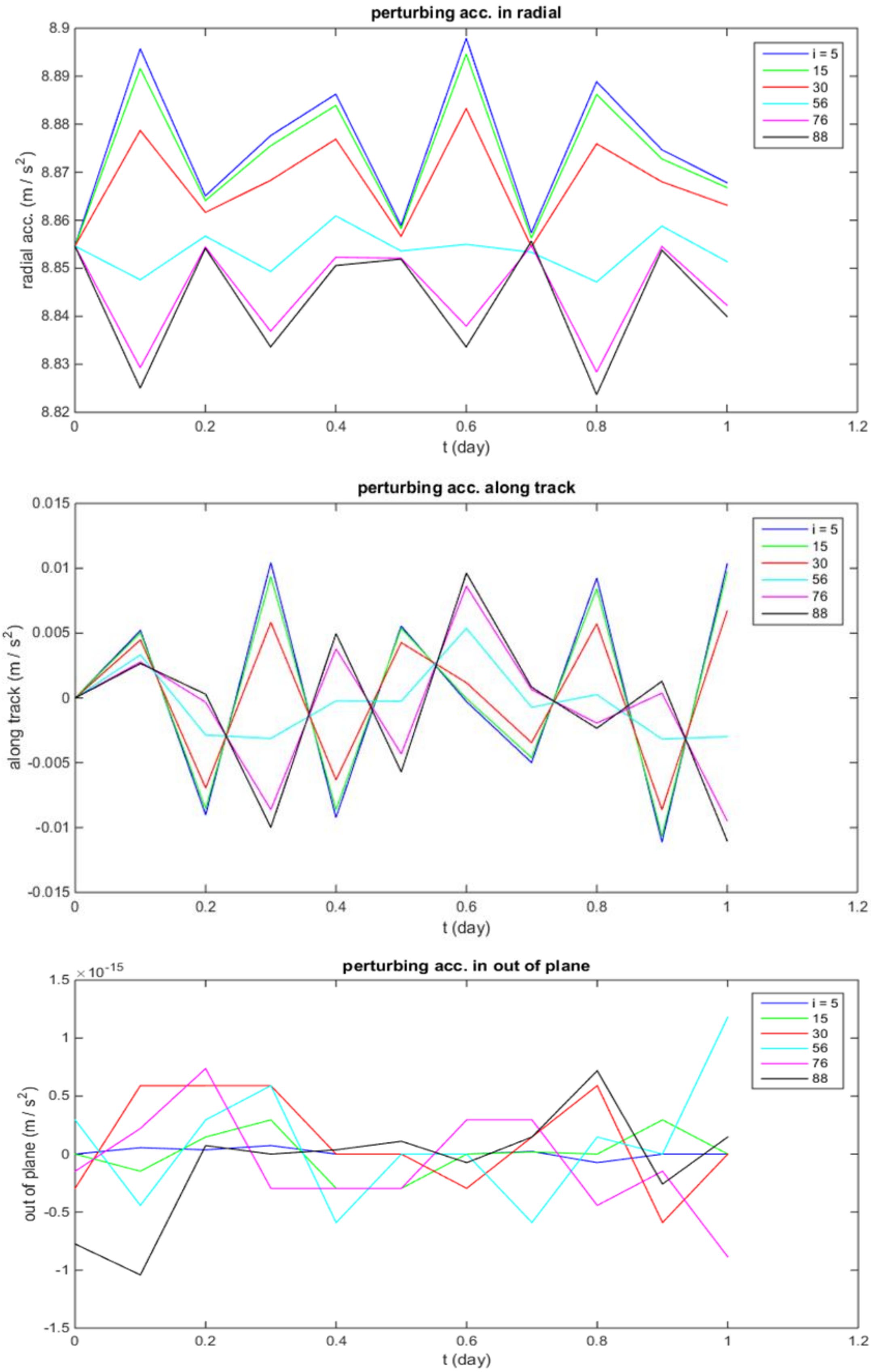

Figure 11. the acceleration coordinates of a low orbits satellites in Earth's oblate gravitational field.

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